Chapter # 6

CULTURAL ONTOLOGY IN MATHEMATICS EDUCATION:
A pedagogical framework for geometry instruction in pre-service teacher training

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ABSTRACT
This study explores integrating historical perspectives into geometry instruction for pre-service mathematics teachers. Traditional curricula portray mathematics as fixed, emphasizing Euclid, Pythagoras, and Thales, while neglecting non-Euclidean contributors like Lobachevsky, Bolyai, and Riemann. Eight third-year students at Achva Academic College engaged in a learning sequence using the Discipline-Culture structure (Tseitlin & Galili, 2005) to examine shared and discipline-specific contexts, as well as contested concepts. They analyzed historical definitions of angles and expanded the DC-Structure with Non-Euclidean axioms. Guided by Cultural Ontology, students recognized mathematics as evolving perspectives. This study employs a mixed-methods approach through triangulation analysis, integrating qualitative research, ontology construction, and historical analysis. Findings showed varied engagement, fostering reflective, informed educators.

Keywords: discipline culture, cultural ontology, meaningful learning, geometry.

1. INTRODUCTION

In today's rapidly evolving world, groundbreaking innovations and societal transformations are unfolding at an unprecedented pace. To not only adapt to these changes but to actively shape them, the younger generation must cultivate a new repertoire of skills and dispositions. Among these are cognitive flexibility, the ability to consolidate and co-create knowledge, and creative problem-solving. This shift reflects a broader transformation in educational philosophy: from the traditional notion of the "educated person" to that of a "culturally engaged individual." Such an individual integrates diverse, and often conflicting, cultural perspectives into their thinking and actions. Because these perspectives differ in values, methodologies, and worldviews, their integration requires a fundamental rethinking of educational content and structure (Bibler, 1991).

In response to this paradigm shift, teacher education should be reimagined to prepare educators who are not only well-versed in disciplinary knowledge but also culturally responsive, adaptable, and equipped to guide students through the complexities of a pluralistic and dynamic world. Mathematics education—particularly in the domain of geometry—often falls short of these aspirations. School curricula tend to present only finalized, "correct" knowledge, omitting the rich historical debates, conceptual tensions, and alternative frameworks that have shaped the discipline. Textbooks typically emphasize canonical figures such as Pythagoras, Euclid, and Thales, while neglecting foundational yet contested thinkers like Lobachevsky, Riemann, and Bolyai. This linear and distilled approach deprives students of the opportunity to appreciate how mathematical knowledge evolves through discourse, controversy, and paradigm shifts. In contrast, meaningful learning requires engagement with conceptual tensions and structured frameworks—an approach exemplified by Galileo's *Dialogues* and Newton's geometrically structured *Principia* (Matthews, 2000; Marton, 2015).

Organizing knowledge is essential for fostering deep understanding (Ausubel, 1968). A well-structured presentation of concepts enables learners to recognize connections, grasp underlying principles, and retain information more effectively. It also supports critical thinking by illuminating relationships, hierarchies, and contrasts between ideas. In educational practice, thoughtful organization transforms complex material into coherent, accessible, and intellectually engaging learning experiences.

The Discipline–Culture (DC) paradigm, introduced by Tseitlin and Galili (2005), offers a transformative framework for reimagining science and mathematics education. At its core lies the philosophical concept of ontology, which serves as a bridge between conventional disciplinary knowledge and a more culturally embedded form of understanding—referred to as Cultural Content Knowledge (CCK, Galili, 2012). This ontological foundation can be computationally implemented through Cultural Ontology, which enables structured and meaningful representations of knowledge. The relationship between the DC paradigm and the DC structure lies in the fact that the DC model—characterized by its triadic architecture of Nucleus–Body–Periphery—serves as the concrete structural framework through which the educational principles of the DC paradigm are implemented and represented.

Inspired by data science, the concept of Multi-Viewpoint Ontologies (MVPo), as articulated by Zhitomirsky-Geffet (2022), offers learners multidimensional access to knowledge through hierarchical, visual, and relational structures. In educational contexts, Cultural Ontologies adopt the same structural model as MVPo; however, the term Cultural Ontology emphasizes the unique character of mathematics and science as expressions of DC structure. When scientific theories are organized within this framework and represented through Cultural Ontologies, they promote deeper conceptual clarity, enhance relevance, and foster engagement with diverse mathematical traditions and theoretical perspectives.

In light of the DC paradigm and the current state of mathematics education in Israel, this study investigates how integrating CCK can make mathematics more meaningful and engaging for students, particularly for pre-service mathematics teachers. For many learners, mathematics is perceived as a dry, technical subject studied primarily for its utilitarian applications. However, conceptual understanding goes beyond rote memorization or mechanical procedures. It involves grasping core ideas, enabling the transfer of knowledge to novel scenarios and its adaptation to varied contexts. Rather than accumulating isolated facts or methods, meaningful learning requires organizing knowledge into a coherent structure—one that defines relationships, establishes hierarchies, and ensures validity and reliability.

This research focuses on the development of Cultural Ontology for geometry, grounded in the theoretical framework of the DC paradigm. The pilot study aims to examine the impact of the CCK approach on the teaching of geometry to pre-service mathematics teachers, enriching their conceptual understanding and enhancing their pedagogical practice.

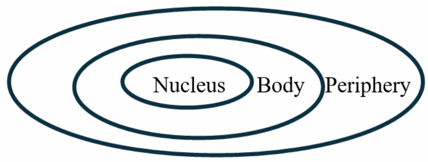
2. BACKGROUND

2.1. Discipline-Culture Approach to Subject Matter a Culture of Rules

Our proposal builds on the paradigm of cultural learning, emphasizing plurality of views and conceptual variation within disciplinary knowledge. Such variation reflects the socially developed nature of knowledge, where discourse incorporates diverse meanings and perspectives. To introduce the cultural aspect while clarifying the status of concepts, Galili and Tseitlin (2005) proposed a special structure for physical theory. A physical theory is seen as an inclusive construct, organizing numerous knowledge elements within a paradigm and domain. They argued that any fundamental theory arranges statements in a centralized structure—nucleus, body, and periphery—forming a quasi-culture.

In this structure (Fig. 1), the nucleus of the theory includes fundamental principles, ontological and epistemological, and the paradigmatic model of the theory. Body elements are all those knowledge elements coherent with the nucleus (theoretical and empirical) and explained by it. The periphery area includes the elements of knowledge in odds with the nucleus (alternative accounts and concepts from other disciplines, old and new, unsolved problems, unexplained phenomena).

Figure 1.
Symbolic representation of areas of knowledge elements within Discipline-Culture structure of a fundamental theory (Tseitlin & Galili, 2005).



It is the periphery that introduces a dialogue between different theories making disciplinary knowledge cultural. It also makes explicit the limits of validity of the particular theory. Thus, while learning the Newtonian account of motion in mechanics, the students become aware of the alternative concepts of motion in earlier accounts by Aristotle and Philoponus as well as the pertinent ideas from relativistic and quantum theories. This way the considered concepts learned obtain *a space of conceptual variation* essential for meaningful learning (Marton, 2015). Our understanding consolidates in comparison with alternatives, discerning the essence of the concept from the differences. Plurality makes the Discipline-Cultural knowledge more stable and stronger than the pure disciplinary univocal one.

2.2. Cultural Content Knowledge for Geometry

This work introduces Cultural Content Knowledge (Galili, 2012) by presenting a historical and conceptual survey of geometry, tracing its evolution from Euclid's classical foundations to the development of non-Euclidean geometry.

The foundations of classical geometry were laid by the ancient Greek mathematician Euclid (circa 300 BCE) in his seminal work *Elements*. In this treatise, Euclid systematically developed geometry based on five postulates. The first four postulates are intuitive and straightforward, but the fifth—known as the parallel postulate—proved to be far more complex and controversial.

Euclid's fifth postulate can be stated as follows: *If a straight line intersects two other straight lines such that the sum of the interior angles on the same side is less than two right angles, then the two lines, if extended indefinitely, meet on that side.* This formulation is less elegant than the others and appears more like a theorem than a fundamental axiom. Over time, many mathematicians attempted to prove the fifth postulate using only the other four, believing it to be derivable rather than independent.

An equivalent and more modern formulation was proposed by John Playfair (1748–1819) *Given a line and a point not on it, there exists exactly one line through the point that does not intersect the given line.* This version became widely accepted and is often used in contemporary discussions of Euclidean geometry.

During the 18th and early 19th centuries, numerous efforts were made to prove the parallel postulate. Girolamo Saccheri (1667–1733) made significant progress by exploring the consequences of denying the postulate. Although he intended to prove its necessity, his work inadvertently laid the groundwork for alternative geometries. His proof contained logical flaws, but it opened the door to new possibilities (Kline, 1985).

Carl Friedrich Gauss (1777–1855), one of the greatest mathematicians of all time, independently discovered the principles of non-Euclidean geometry. However, fearing backlash from the mathematical establishment, he chose not to publish his findings. It was left to two bold thinkers—Nikolai Lobachevsky (1793–1856) in Russia and János Bolyai (1802–1860) in Hungary—to formally develop and publish the first non-Euclidean geometry, now known as hyperbolic geometry. Lobachevsky published his work in 1825, and Bolyai followed in 1832 (Kline, 1985).

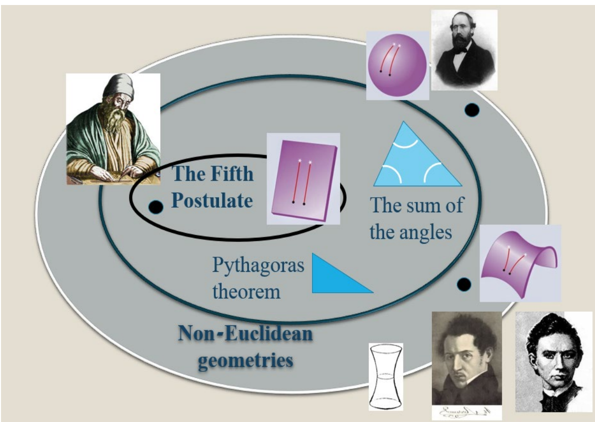
In hyperbolic geometry, the parallel postulate is replaced with the assertion that: *Given a line and a point not on it, there exist at least two distinct lines through the point that do not intersect the given line.* This radical departure from Euclidean principles led to a consistent and coherent geometric system. Eventually, models of hyperbolic geometry were constructed within Euclidean geometry itself, demonstrating that its consistency is equivalent to that of Euclidean geometry.

Later, Georg Friedrich Riemann (1826–1866), a student of Gauss, introduced another form of non-Euclidean geometry—spherical or elliptic geometry—in which all lines (great circles) intersect. Riemann's work extended the concept of geometry to curved surfaces and laid the foundation for Riemannian geometry, a general framework that encompasses Euclidean, hyperbolic, and spherical geometries as special cases (Kline, 1985).

Riemann's insights were revolutionary, influencing not only pure mathematics but also theoretical physics. His geometric framework became a cornerstone of Einstein's general theory of relativity, where the curvature of spacetime replaces the flatness assumed in Euclidean geometry.

The data can be presented in the form of DC structure in the following Figure 2.

Figure 2.
DC of Euclidean geometry vs Non-Euclidean geometries.



2.3. Curricular Infrastructure based on Cultural Ontology

In the field of Information Science, scholars increasingly emphasize the need to design knowledge systems that are not only logically coherent but also ethically inclusive. This dual focus is particularly critical in contexts where diverse perspectives and conflicting viewpoints coexist. Within this discourse, Cultural Ontology emerges as a structured representation of mathematical knowledge that integrates both consensual and culturally specific elements. It functions simultaneously as a formal framework and a pedagogical tool, enabling the organization, comparison, and contextualization of concepts across different mathematical traditions. Such an approach enriches the learning process by highlighting connections and contrasts between ideas, thereby fostering deeper understanding.

Zhitomirsky-Geffet and Avidan (2022) advance this vision by proposing a framework for systematically identifying and classifying inconsistencies in Multi-Viewpoint Ontologies (MVPo). Their model, grounded in description logic and set theory, analyzes eight logical structures of ontological statements and applies them to case studies in nutrition and geometry. Findings reveal that inconsistencies often arise from three recurring structures (S2, S3, S5). To address these, the authors introduce the concept of a "validity scope," a contextual domain—whether theoretical, demographic, or situational—within which each statement holds true. This innovation allows heterogeneous knowledge to be integrated into a logically consistent system, with practical implications for intelligent applications such as decision-support tools and semantic search engines.

Parallel to this, the CLARE framework for climate-resilient development (Cochrane et al., 2023) outlines five functional areas: research collaboration, knowledge management, monitoring and learning, research uptake, and adaptive management. By emphasizing integrated implementation, CLARE fosters continuous learning, adaptive responsiveness, and effective impact measurement. These principles resonate strongly with the dynamic nature of multi-perspective knowledge systems, underscoring the importance of flexibility and inclusivity.

Ethical dimensions are further explored in Zhitomirsky-Geffet's (2022) care ethics-based framework for evaluating multi-perspective knowledge organization systems. This model introduces criteria such as viewpoint identification, fair representation, ethical tagging, transparency, and moral balance. A complementary framework (Zhitomirsky-Geffet & Hajibayova, 2020) expands on these ideas by incorporating ethical-logical principles, advocating transparent classification logic and moral integration of diverse perspectives. Together, these models challenge traditional notions of neutrality in information systems and promote human-centered approaches.

Finally, Zhitomirsky-Geffet and Avidan (2022) present a two-phase crowdsourcing methodology for constructing multiviewpoint ontologies. Experts first extract scientific content, followed by non-expert participants classifying 750 statements as true, opinion, or error. Remarkably, participants achieved over 90% accuracy by evaluating what others might think rather than expressing personal views. This demonstrates the potential of structured collective intelligence in building robust, inclusive ontologies.

Collectively, these studies contribute to a unified vision for future knowledge systems—one that integrates logical rigor, ethical responsibility, and participatory inclusivity. Within mathematics education, MVPo as Cultural Ontology exemplifies how thoughtful knowledge organization enhances meaningful learning, supports critical thinking, and makes complex material more coherent and accessible.

3. RESEARCH

3.1. Research Design and Methods

This study applies the Discipline–Culture (DC) paradigm to the teaching of geometry in schools, examining both Euclidean and non-Euclidean traditions through a triadic framework of nucleus, body, and periphery. While the nucleus varies across traditions, its shared pedagogical goal is to foster conceptual understanding by comparing core knowledge with peripheral alternatives. The project includes four components. First, instructional materials are developed, grounded in the DC paradigm and Cultural Ontology, to enhance students' comprehension of geometrical concepts. Second, a course is designed for pre-service mathematics teachers, introducing them to the DC paradigm, cultural ontology construction, and strategies for teaching geometry through this lens. Third, pilot teacher training programs are conducted in colleges of education, integrating the DC paradigm into teacher preparation and evaluating its effectiveness. Finally, assessment tools are created to measure the quality and impact of instructional materials and ontologies, focusing on their contribution to conceptual understanding among pre-service teachers.

3.2. Research Questions

The study is guided by the following research questions:

1. To what extent does the DC paradigm promote conceptual understanding in geometry?
2. What is the quality of conceptual understanding of geometrical concepts as facilitated by the DC paradigm?
3. How do pre-service high school mathematics teachers' views on the nature of the knowledge they learn evolve through engagement with cultural content knowledge?

These questions aim to explore both the theoretical and practical implications of applying the DC paradigm to mathematics education, particularly in the context of geometry.

3.3. Methodology

This study employs a mixed-methods approach through triangulation analysis, integrating qualitative research, ontology construction, and the historical analysis of mathematics.

3.3.1. Empirical Data Collection and Analysis

This research adopts a qualitative methodology, grounded in criteria-focused patterns that emphasize analytical rigor over intuitive interpretation. Researchers rely on a system of external criteria—such as theoretical frameworks, guiding principles, and categorical structures—throughout the study. While this approach shares certain features with positivistic traditions in quantitative research, it remains distinct in its philosophical orientation and interpretive depth (Shkedi, 2011).

The criteria-based methodology integrates both analytical and impressionistic resources. Within the qualitative paradigm, researchers aim to articulate phenomena in verbal and conceptual terms, often engaging with philosophical dimensions of meaning. This methodological framework includes all the core components characteristic of qualitative inquiry, as outlined by Shkedi (2011), and supports a nuanced exploration of educational phenomena through culturally and epistemologically informed lenses.

3.3.2. Cultural Ontology Construction

This study introduces a methodology for constructing a cultural ontology in geometry, grounded in the Discipline–Culture (DC) structure proposed by Galili and Tseitlin (2005). The central aim is to represent disciplinary knowledge in a way that integrates both consensual and culturally diverse elements, thereby promoting deeper understanding through comparative analysis.

The first phase involves the construction of Single-Theory Ontologies. Using formal structures such as OMDoc (Lange, 2013), each ontology is built around a nucleus, representing the foundational principles of the theory, and a body, comprising concepts derived from the nucleus. Semantic relationships—including hierarchy, inclusion, and entailment—define the connections between concepts, while each element is annotated according to its structural role within the theory.

The second phase focuses on the Integration of Cultural and Peripheral Knowledge. Ontologies are expanded to include peripheral elements, with shared concepts across theories marked as consensual and theory-specific elements labeled cultural. This distinction reflects historical, epistemological, and sociocultural contexts. To account for varying conditions of validity, a new ontological class—Validity Scope—is introduced (Zhitomirsky-Geffet, 2019; Zhitomirsky-Geffet & Avidan, 2022). Each theory is linked to a specific scope, allowing knowledge to be represented as context-dependent.

The resulting Cultural Ontology provides a formal structure of geometric knowledge that incorporates consensual and cultural elements, inter-theory relationships, and contextual validity. This model advances the DC paradigm's goal of fostering meaningful learning by presenting mathematics as a dynamic, culturally embedded discipline.

3.4. Empirical Settings and Results

3.4.1. First Stage Questionnaire

Eight third-year undergraduates in Achva Academic College, Israel —future junior high mathematics teachers—were asked to define mathematical contexts shared by mathematics and physics (e.g., derivatives), contexts relevant only to one discipline (e.g., irrationality in mathematics), and content lacking consensus across different mathematical theories (e.g., parallel lines). The answers they provide show that there was no awareness of the plurality of mathematical theories, variation of definitions and diachronic discourse.

3.4.2. Second Stage Learning

1) Eight third-year undergraduates learned about various historical definitions of the concept of angles (Barabash, 2016) and how these definitions influence current mathematics curricula and textbooks at all school levels.

2) They were introduced to the DC paradigm and CCK examples (Galili, 2014; Vinitsky- Pinsky & Galili, 2014a, 2014b; Levrini et al., 2014; Vinitsky- Pinsky et al., 2025b).

3) They were introduced to the network of inter-linked cultural ontologies (NCO), especially the example of geometry ontology developed by their lecturer (Zhitomirsky-Geffet, 2019). The lecturer had to write a reflection on the process she was going through with students. The students were surprised since they were used to a polyphonic representation of the disciplinary content.

3.4.3. Third Stage Implementation

Eight third-year undergraduates were asked to apply the knowledge they were exposed to by doing a summary assignment in the course. The final assignment involved expanding a Cultural Ontology in geometry by constructing the section on Spherical Geometry. Building on the existing framework of Euclidean, Synthetic, and Lobachevskian Geometry, the task required identifying fundamental concepts of Spherical Geometry, documenting its axioms, and comparing propositions across different geometries. Specifically, we analyzed which propositions hold in both Synthetic and Spherical Geometry, which are exclusive to each, and explored logical relationships such as “derives-from,” inclusion, and overlap. This process deepened our understanding of how geometric systems differ and relate, and helped develop a structured, comparative approach to teaching geometry through conceptual and cultural perspectives.

The students were divided into three work groups for this assignment, with group sizes of 3, 3, and 2 (Vinitsky-Pinsky et. al, 2025a). All the groups successfully completed the task, though they noted that it posed significant challenges from a mathematical perspective. To overcome these difficulties, they frequently consulted a variety of academic sources. Figure 3 demonstrates the comparison suggested by Work Group number 2.

The students' performance in this assignment highlighted how DC-based materials foster dialogue between ideas, encouraging learners to engage in the discourse of geometric knowledge.

Figure 3.
Group 2, Euclidean Geometry vs. Spherical Geometry.

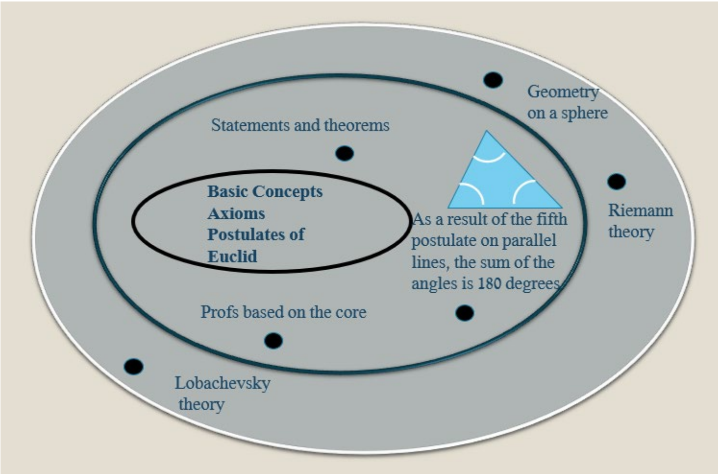
Aspect	Euclidean Geometry	Spherical Geometry
Fundamental Concepts	Point, line, segment, angle, circle, congruence	Point, great circles, arcs, angle, congruence
Axioms	Includes the Parallel Postulate	No parallel arcs; infinite perpendiculars
Triangle Angle Sum	Exactly 180°	Greater than 180°
Congruence Theorems	SAS, ASA, SSS	SAS, ASA, SSS, AAA
Similarity	Similar triangles may not be congruent	No similar triangles—only congruent
Betweenness	Defined: one point lies between two others	Undefined
Simplest Polygon	Triangle	Digon (2 sides)

Below is an explanation from one of the students about how they utilized the DC structure to organize their understanding:

> “*We can organize the knowledge for the students in the way that when they learn something new (a definition or theorem) we check where it is placed (in DC) and understand how it is connected to the principles…. If we change a definition of a principle we get another geometry – Non-Euclidean…” (participant 1)*

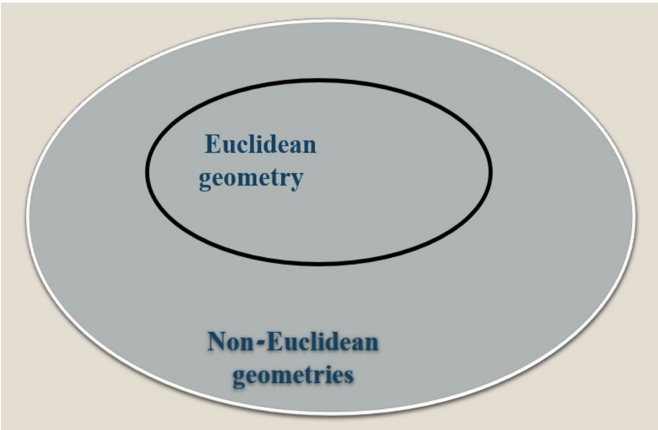
> *“The nucleus-basic concepts of the Euclidean geometry. The body – sum of triangle angles 180 degrees. The periphery - spherical geometry, "Riemannian geometry...” (participant 5, Figure 4)*

Figure 4.
Participant 5, DC-structure of geometry.



Of the eight participants, only one did not successfully represent the DC framework; as illustrated in Figure 5, the body was omitted.

Figure 5.
Participant 8, DC-structure of geometry.



Correct answers and successful performance on the assignment reflect the pre-service teachers' growing conceptual understanding of geometry. The Cultural Ontology, grounded in diverse theories and approaches, exposed them to multiple definitions and connections across Euclidean, hyperbolic, and spherical geometries. These differences, together with the hierarchical structure of the discipline, became visible through the ontology database. Based on their responses, pre-service teachers experienced a noticeable shift in how they perceived the nature of mathematical knowledge.

For instance, when teaching parallel lines, the Cultural Ontology framework allowed Euclid's postulate to be presented alongside its alternatives in non-Euclidean systems. This comparative approach encouraged students to reflect on underlying assumptions and deepened their appreciation of mathematical reasoning. The lecturer observed that once students began constructing the ontology themselves, their engagement increased significantly. Initially hesitant, they gradually embraced the complexity of comparing geometrical systems and started asking profound questions about mathematical truth. Their reflections revealed an emerging awareness of mathematics as dialogical and culturally situated.

This study demonstrates that integrating the Discipline–Culture paradigm with Cultural Ontology construction enhances conceptual understanding by encouraging students to view mathematics not as static truths but as a dynamic, culturally embedded discipline. Building and comparing ontologies functioned both as a pedagogical strategy and a reflective tool, enabling participants to articulate epistemological distinctions and reconsider assumptions. These findings suggest that embedding cultural and historical perspectives into teacher education fosters deeper learning and prepares future educators to teach mathematics with greater clarity and cultural awareness.

The Linking Research Questions to Data and Findings are presented in Table 1.

Table 1.
Linking Research Questions to Data and Findings.

Research Question	Type of Data Presented	Key Findings	Insights
1. To what extent does the DC paradigm promote conceptual understanding in geometry?	Final assignments, ontology construction, geometry comparisons	Students identified differences between geometries, classified concepts using the DC structure, and explained interconnections	Demonstrated deep conceptual understanding through structured knowledge organization and comparative analysis
2. What is the quality of conceptual understanding of geometrical concepts as facilitated by the DC paradigm?	Reflections, group work, student quotes	Some participants articulated distinctions between systems and axioms; others struggled with epistemological complexity	Revealed a range of understanding levels, with a shift from procedural to structural thinking
3. How do pre-service teachers' views on the nature of mathematical knowledge evolve through engagement with Cultural Content Knowledge?	Reactions to historical content, exposure to polyphonic representations	Students expressed surprise and curiosity, recognizing mathematics as culturally embedded and dynamic	Shifted from viewing math as fixed and universal to seeing it as pluralistic and evolving

4. FUTURE RESEARCH DIRECTIONS

The findings of this study highlight several promising directions for future research in mathematics education, particularly in preparing teachers. First, exploring ontological thinking in teacher training may reveal how engagement with cultural ontologies enhances conceptual flexibility and pedagogical strategies. Longitudinal studies could show whether such exposure leads to lasting changes in teaching practices and curriculum design. Second, integrating mathematics and physics offers opportunities to present cohesive, culturally enriched content, addressing gaps in awareness of shared and divergent contexts. Third, students' surprise at polyphonic representations suggests investigating how multiple voices in curricula influence engagement and critical thinking. Finally, research on scalability could extend cultural ontologies to algebra, calculus, and diverse contexts.

5. CONCLUSION

Although the mathematical component of the course presented a significant challenge, the pre-service teachers gained substantial pedagogical and conceptual benefits from the training grounded in the Discipline–Culture paradigm.

The unique contribution of this chapter lies in its integration of philosophical theory, formal knowledge representation, and practical pedagogy. By bridging Cultural Ontology with teacher education, it offers a replicable model for fostering epistemological awareness and conceptual depth in mathematics instruction.

First, that pre-service teachers engaged in an open-minded, culturally enriched learning environment that contextualized knowledge, enabling them to see the broader picture and recognize practical applications of theoretical concepts across related theories.

Second, through ontology mining, they explored both intra- and inter-disciplinary connections, deepening their understanding of theoretical foundations, historical development, and cross-theory relationships.

Third, exposure to a variety of theories and perspectives allowed them to appreciate the underlying reasons for similarities and differences among theories, fostering a more nuanced and critical view of mathematical knowledge.

Finally, the integration of Cultural Content Knowledge into mathematical instruction empowered pre-service teachers to design learning experiences that are both inclusive and contextually responsive. By exploring the diverse historical trajectories and epistemological foundations of mathematical thought, they cultivated the ability to teach mathematics in ways that move beyond mere procedural fluency—toward deeper conceptual understanding and cultural relevance. This pedagogical evolution reflects a broader shift in educational philosophy: from the static ideal of the "educated person" to the more dynamic and adaptive concept of the "culturally engaged individual." Such individuals are not only informed but also attuned to the complexities of cultural diversity, capable of integrating multiple perspectives into their thinking and practice within an increasingly interconnected world.

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