# Chapter # 12

# A TRANSFORMATIVE INSTRUCTION MODEL AND TASK-ORIENTED LEARNING

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**ABSTRACT**
This study is analytical research that examines a theoretical approach to transformative pedagogy (TP) and transformative learning (TL) in order to conduct a logical deductive construction of the transformative instruction model (TI model). The study aims to enhance the understanding and effectiveness of the interplay between teaching content and learning in the form of requires for the high-quality teaching of students in the cognitive development of mathematics. It also analyzes the conditions for transformative teaching and learning in a task-oriented context. The crucial psychological outcomes focus on the domains of TP and their interpretation within the context of Mezirow's TL theory, emphasizing an instrumental, dialogic and self-reflective learning discourse, as well as meaning schemes in learning. The methodological approach is based on a systematic deductive analysis. The analytical results as a conceptualization and an analytical construction of a TI model related to transformative teaching and learning algebraic concepts. The TI model is the result of extending theoretical approaches to TP and TL, as well as logical connections to research about the formation of algebraic concepts. Benefits of the TI model is a new knowledge about an effective teaching of algebra, where students' algebra learning, and knowledge development is crucial.

*Keywords:* transformative mathematical instruction, transformative algebra learning, task-oriented teaching, algebraic concepts.

## 1. INTRODUCTION

Teachers play a crucial role in students' learning and socio-emotional development, which requires a specialized knowledge base informed by research and practical insights (Guerriero, 2017). Effective teaching, that combines expertise in mathematics with an understanding of students' perspectives, creates an engaging learning process (Schwarz & Kaiser, 2019).

Despite the established principles in school mathematics (NCTM, 2000), empirical research suggests there is a need for teachers to re-evaluate their understanding of teaching mathematics. This re-evaluation is essential for achieving a deeper understanding that correlates with improved student achievement (Ulferts, 2021). Continuous learning is closely tied to enhanced overall quality of mathematical instruction (Karpov & Haywood, 1998; Daniels, 2003).

In Sweden, recognition of teachers' pivotal contributions has led to a national professional development initiative spearheaded by the Swedish Agency for Education (2012). This initiative aims to empower teachers with a structured, research-informed approach to professional growth that emphasizes the enhancement of education quality. However, despite this initiative, Sweden faces challenges in mathematics performance compared to other countries, as indicated by international studies (e.g., Mullis, Martin, Foy, Kelly, & Fishbein, 2020; OECD, 2023).

An acknowledgment of the central role of teaching and continuous learning among educators is essential for enhancing the overall quality of education (Ma, 1999; Ball & Bass, 2000). Moreover, empirical research underscores the need for teachers to develop their understanding of reflective teaching practice, a crucial step in effectively improving student achievement (Scheiner, Montes, Godino, Carrillo, & Pino-Fan, 2019). To address these challenges, this study explores transformative pedagogy (TP) in combination with transformative learning (TL) as a viable alternative for teachers to enhance their approach to teaching mathematics that emphasizes the ongoing significance of teachers' professional development.

## 2. BACKGROUND

### 2.1. Why the Transformative Instruction Model (TI Model)?

This section outlines five arguments for the need to construct a new model for transformative instruction (TI model) that employs TP/TL learning. This will be further explored in the next section.

Firstly, empirical evidence of teachers' mathematical knowledge related to their teaching has led to discussions on improving the quality of instruction in mathematics education. There has been much research about this topic, with various theories emerging since Shulman's (1986) work. However, empirical evidence on the connection between a teacher's mathematical knowledge and its application in instruction remains limited, as highlighted by Ronda and Adler (2019). There is a need for more theoretical and empirical studies to address this gap and provide concrete insights.

Secondly, the studies emphasize the importance of teachers' having professionally specific knowledge for teaching mathematics in order for their students to improve (Ball, Hill, & Bass, 2005). However, the exact type of knowledge needed–and whether this is just about basic skills is a complex question. At the same time the nature of teachers' specific professional knowledge in relation to their mathematical knowledge plays a crucial role in students' learning and skills development. Effective learning requires finding a balance between conceptual and procedural knowledge, with conceptual understanding being crucial for students to progress (Smith, Bill, & Raith, 2018). Research continues to discuss the nature and extent of teachers' professionally specific mathematical knowledge (e.g., Hammer & Ufer, 2003). While various studies have indicated its importance for improved student achievement, the exact type of expertise needed, whether basic or complex skills, professionally specific knowledge remains unknown (Ball et al. 2005). This ongoing research emphasizes the continuous effort to identify the optimal type of professional mathematics knowledge for fostering effective teaching and student learning.

Thirdly, as mathematics is an inherently abstract subject, it requires high-quality teaching that will benefit students' cognitive development. Here, teaching quality is critical (e.g., Carter, 2024). According to Vygotsky (1986), students' mental development depends on the interaction of a mediating agent, such as a teacher, with the learning environment. Teaching mathematical concepts in line with Vygotsky's constructivist approach (Vygotsky & Rieber, 1998) is effective when the instruction is well-structured, focused on the learning process, and involves dynamic mental collaboration between the teaching content and the students (Venkat, 2021). The "ambitious instruction" MPM model highlights the importance of students' ability to generalize in the learning process, guiding teaching structure to support the understanding of content (Venkat & Askew, 2018). Teachers' knowledge of mathematics that is described in teaching plays a crucial role in improving students' ability to learn mathematical concepts (Brown, 2011) and their in-depth understanding of the nature of students' learning is crucial for their students to acquire mathematical skills (Venkat, Askew, Watson, & Mason, 2019). This analytical research demonstrates how teachers can organize task-oriented teaching using mediating tools, such as tasks in terms of artifacts.

Fourthly, the mathematics learning process comprises three components: (1) prior knowledge; (2) expanding prior knowledge in a new learning process; and (3) reproducing new knowledge through reasoning and reflection (Nickerson, 2012). This encapsulates the core of learning and its process components, emphasizing teaching as a tool for developing learning through content. Critical reflection and thinking are crucial for teaching and learning opportunities (Dirkx, Mezirow, & Granton, 2006), in keeping with sociocultural views on mathematics learning in the conceptual learning framework about learning activity (Zuckerman, 2003). This argument led that needs to development new instructional model is referred to as *analytical research*.

Fifthly, this widely used framework in mathematics education comprises three core aspects: task analysis, action planning, and reflection, the latter of which encompasses, for example, learners' self-evaluation and understanding of alternative viewpoints (Olina & Sullivan, 2004). Crucially, it underscores the key step in the learning process, knowing mathematics through critical reflection (Liu, 2015). More research about mathematical instruction as an interplay between teaching and learning could incorporate teachers choices and analyses of learning tasks, planning teaching based on students' learning, and reflective practice including learners' self-evaluation and understanding of alternative mathematical viewpoints (Sakai et al., 2024). TP and TL concepts serve as design frameworks, addressing content, the learning process, critical thinking, and reflection (Johnson & Olanoff, 2020).

The above-cited studies provide a conceptualization and argumentation of theory-based student-oriented teaching. They analyze student learning and emphasize a key aspect related to TL. TL highlights the importance of exploring and understanding the learning process, particularly in teaching school tasks (e.g., exercises and assignments), through modified pedagogical design. The study of TP and TL is in line with research on professional development (Mezirow, 2003), experiences, and practices (Mezirow, 1997; Fleming, 2021), as well as current students' achievements in an analytical context.

This chapter on the incorporation of TP and TL theoretical tools into the TI model includes components as addressed and related to arguments 1–5 and an appropriate understanding of the meaning of mathematical instruction that previse future of design for teaching and learning mathematics (algebra).

The purpose of this analytical study is to analyze the outcome of transformative pedagogy (TP) and transformative learning (TL) in task-oriented teaching. It presents a theoretically based model for transformative instruction (TI model) that is specifically focused on enhancing students' learning of algebra. The research questions are as follows:

RQ1: How can the relationship between transformative pedagogy (TP) and transformative learning (TL) effectively enhance teaching focused on students' learning of mathematics (TI model)?

RQ2: What aspects of a TI model should be included in an appropriate algebra-specific didactic approach for transformative teaching and learning?

The TP and TL theoretical framework proposed in this study aims to stimulate discussion on the structural aspects of task-oriented teaching, providing a conceptually grounded model for transformative instruction (the TI model).

## 3. THEORETICAL OUTLINES

To enhance understanding of TP, a framework is presented that serves as a comprehensive design model, guiding the development of teaching and learning outcomes across philosophical, psychological, and social dimensions (Farren, 2016). This framework involves an interplay between the components of TP and conceptual meaning. The components encompass aspects that are crucial for the development of teachers and students, including identity, beliefs and attitudes, moral-ethical values, socio-affective factors, communication, (meta)cognition, school, and society. In this context, conceptual meaning encompasses reflective practices, reflective teaching, analysis of learning, and supportive learning, and leads to the development of instruction through the transformation of teachers and students via reflection and self-reflection (Farren, 2016). In this transformative process, various forms of knowledge (both personal and subject-content-specific) interact, leading to the transfer and utilization of knowledge for solving subject-specific and social-specific problems (Chapman & Guerra, 2016).

The theoretical aspects of TP, particularly within (meta)cognition, significantly impact reflective practices and the reflective teacher, emphasizing the role of the teacher in analyzing learners' needs and fostering autonomy (Farren, 2016; Fleming, 2022). TP creates conditions for practices in which teachers' knowledge of learning significantly enhances their potential for development, leading to transformative learning experiences for their students. The development of students' mathematical reflections into interaction of metacognitive and self-regulatory requirements refers to students' learning of mathematics through reflections and mathematical reasoning (Thompson, 1996). These mathematical reflections are employed under the framework of transformative teaching through various mathematical tasks related to subject-specific content to ensure an effective and profound understanding of mathematics (Hassi & Laursen, 2015). Mathematical activities that focus on a clear learning goal related to the proper planning of tasks, teachers' evaluation of students' achievements, and reflection on the students learning process provide the necessary conditions for students learning performance (Kinard & Kozulin, 2008) and transformative learning (Adams et al., 2015).

This pedagogical approach helps in the comprehension of the key components that are intrinsic to the TP concept. In keeping with TP, teachers construct knowledge through their practices and collaboration, fostering critical reflections and intentions for self-development and improvement in teaching and learning (Luitel, 2013).

An analysis of TP using Farren's framework (2016) underscores the fact that TP models, including all designs, should incorporate an appropriate subject-specific didactic approach, particularly in mathematics education. Teachers can leverage TP domains to develop their instructional practices. Systematic instruction (Smith, Saez, & Doabler, 2018) involves intentionally designing and delivering tasks in order to develop students' mathematical knowledge towards achieving specific learning outcomes and individual cognitive development. A response to RQ1 regarding the TP concept emphasizes a focus on phenomena such as knowing, reflective teaching, reflective practice, knowledge of learning, content knowledge, acts of reflections, and self-reflection. The key point in TP pedagogy' is that teachers' content knowledge is continuously developed through pedagogical reflections and reasoning regarding the reconstruction of the teaching experience, planning, and adapting teaching with regard to students' learning (Johnson & Olanoff, 2020).

For a deeper understanding of active teaching that could be beneficial to students' learning of mathematics, a follow-up TP with a theoretical analysis of TL theory will provide additional insights and knowledge about mathematical instruction and one time more knowledge for transformative teaching. This means that the knowledge and learning process could be more successfully mastered to enhance the understanding of transformative pedagogy, specifically related to the transformative learning of mathematics actions and the practice of learning.

This analysis enhances the understanding of TP, specifically in terms of TL actions and learning practices. The interpretation and development of the TP framework using the TL approach highlighted in this study underlines the conceptual elements for the development of mathematical instruction with regard to content-based knowledge transformation and students constructing of mathematical knowledge.

## 4. THEORETICAL EXPANSION THROUGH THE TL APPROACH

In expanding upon the TP theoretical framework, Mezirow's learning-oriented TL theory serves as the starting point for this study (Mezirow, 1985). Of particular interest are the various types of task-oriented learning, including instrumental, dialogic, and self-reflective learning (Figure 1). In instrumental task-oriented learning, learners seek the best ways to grasp a concept. Dialogic learning explores the optimal time and place for learning, and self-reflective learning questions the purpose of acquiring content.

These learning processes lead to transformation through three types of learning known as “meaning schemes.” In the first “meaning scheme” (leaning within meaning schemes, see Figure 1) process, learners expand upon, complete, and repeat what they have already learned (prior knowledge). This means a conceptual systematization and expansion of students' prior knowledge that has relevance for learning a new mathematical content. The second “meaning scheme” (learning new meaning schemes) process relates to the organizing of students' learning of new content. The third “meaning scheme” (learning through meaning schemes) process is learning by generalizing new mathematical content through task-oriented context. The transformative teaching in this fazes moving to teachers' analysis and systematization of students' mathematical learning achievements, to create written documentation in order to use it for the future planning of teaching. The crucial key in the documentation is the students' learning, challenges and misconceptions that are identified through the redefinition of these challenges and misconceptions. Transformative teaching and learning occur through task-oriented mathematical content and students' critical self-reflection supported by teaching (Kitchenham, 2008; Baumgartner, 2012).

The conceptual transfer from the first to the third “meaning scheme” process depends on the types of learning, such as instrumental, dialogic and self-reflective. I instrumental type of learning is main connected to students' learning from a procedural respective and is particularly impacted by the teachers' step –by –step instructions. The significance of instrumental learning is the limited space it offers for learners' own reflections, explanation, and mathematical reasoning. According to Mezirow (2008), dialogic learning is another type of learning and extends instrumental task-oriented learning. This is a key focus on students' verbalization, reflections and mathematical reasoning. Dialogic type of learning in a mathematical context concern students' active verbalizing of methods or concept outcomes as discussed with their peers (Taylor & Cranton, 2012). The conceptualization of self-reflective learning in the TL framework plays a key role in students' reflective learning by critical thinking, and subject-specific reflections (Weimer, 2013).

The TL framework describes conceptual meaning as different kinds of reflection components associated with three types of learning processes (“meaning schemes”). The components are content, process and premise reflection that led to straightforward and profound knowledge transformation (see Figure 1). In analyzing the TL theoretical framework be clear significant of follow main theoretical points for transformative teaching and learning as types of learning and task-oriented learning inclusive reflection components and it's considered to develop an optimal conceptual model for transformative instruction.

## 5. ANALYTICAL FINDINGS: TRANSFORMATIVE INSTRUCTION, TI MODEL

Analytical tracking of the relationship between TP and TL theories leads to a logical consequence for effectively enhancing transformative teaching focused on transformative students' learning of mathematics in a task-oriented context. How can the relationship between transformative pedagogy theory (TP) and transformative learning theory (TL) effectively enhance teaching that is focused on students' learning of mathematics (RQ1), as depicted in Figure 1?

*Figure 1.*
*Transformative mathematical instruction model (TI model).*

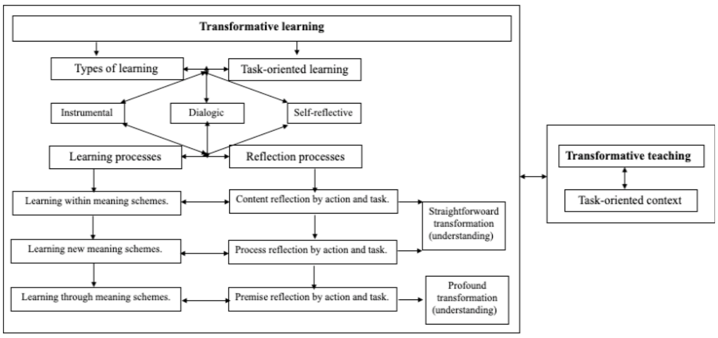


Figure 1 illustrates the development of a theoretical framework for TP concerning transformative task-oriented teaching designed as a TI model for transformative mathematical instruction through the conceptual significance of TL regarding psychological positions for learners' transformative learning. The TI context also includes TP perspectives and emphasizes profound, reciprocal transformations for both teachers and students, with reflection as a key aspect of self-reflection and reflective teaching practices. According to the TI model, the development of teachers' effective practice (teaching mathematics) depends on understanding the major constituents of transformative teaching with a focus on the learners' learning, specifically the constituents of the TI model. The transformation learning process is illustrated in the TI model as the relationship between teaching subject-activities-content (types of learning) and learning processes (meaning schemes) (Figure 1). The orientation to learning is an important conceptual point in the TI model and highlights its multifaceted nature, with transformation resulting from continuity (from prior knowledge to the generalization and application of knowledge) and the connection between the three types of learning. The TI model (Figure 1) demonstrates how teachers can organize task-oriented learning at both straightforward and more ambitious levels, in order to enhance the students' knowledge and understanding. The connection between the TI model (Figure 1) and mathematics-specific-content supports teachers in their practice, emphasizing the value of pedagogical design to mathematics education—subject content and learning.

The starting point for expanding and developing the TI theoretical framework, centered around the concept of transformative learning, is Mezirow's transformative learning theory. This theory is learners-centered and includes ideas and perspectives about the learning process and reflection as discourse that are key to making sense of a students' development (Mezirow, 1985). Specifically, a key component of the TI model is the exploration of different types of task-oriented learning connected to instrumental, dialogic, and self-reflective learning. This finding, which resulted in the creation of a transformative instruction (TI) model, provides a total overview of how to teach learners mathematics, as well as how task-oriented teaching can interact with the transformative cognitive development of students through learning as meaning schemes and different kinds of reflection. The logical combinations of different types of learning (task-oriented learning), the conceptual meaning of learning and reflection processes, are an important part of students' straightforward and profound understanding, as well as their transformative understanding of teaching mathematical content. The primary significance of the TI model lies in transformative teaching and educational practices through task-oriented learning as depicted in Figure 1.

Concerning RQ2 about the aspects of a TI model that should be included in an appropriate algebra-specific didactic approach for transformative teaching and learning, such a model can provide conceptual meaning to different types of learning. In the case of instrumental task-oriented learning, learners inquire about how they can best grasp an algebraic concept; in the case of dialogic task-oriented learning they explore when and where this learning could optimally take place, and in the case of self-reflective learning, they question why they are acquiring algebra-specific content. It is crucial to note that, regardless of the case, the learning process involves three types of learning that lead to transformation. These three types of learning, which are called "meaning schemes", are also an important part of a TI model related to students learning of algebra-specific content, i.e., *the first learning* process (learning within meaning schemes) involves learners working with what they have already learned (know) by expanding, completing, and repeating mathematical concepts and operations. *The second learning* process (learning new meaning schemes) is based on students being able to verbalize the algebraic methods or concept outcomes being discussed with other students and teachers. *The third type of learning* (learning through meaning transformation) is based on students' analyzing their own knowledge development of algebra content. A crucial point in students' learning of algebra through these diverse processes is that challenges, problems, and misconceptions can only be identified through the redefinition of such challenges and misconceptions in arithmetic. Transformation and transformative learning of algebra occur through students' critical self-reflection with the support of teachers during the learning process ("meaning scheme") of both prior knowledge of arithmetic and new algebra knowledge. In terms of the transformative teaching of algebra and the TI model, teachers' reflections play a crucial role in students transformative learning of algebraic concepts. Specifically, teachers analyze their students' work, systematize this information, and use it for future planning of classes.

As regards mathematical activities and algebra content, the key point of the TI model (Figure 1) is that the necessary prerequisites for TI include teachers' self-reflection about teaching and students' learning, motivation for professional development, and readiness for self-development and teaching improvement, as well as a continuous commitment to teaching algebra through the application of their own previous mathematical (algebra) knowledge of teaching. This suggests that as a transformative measure in learning, the TI model is influenced by these prerequisites and the teacher's perspective on their students' learning of algebra.

## 5.1. TI Model, Algebra Content and Task-Oriented Learning

While staying focused on the development of teaching practice as transformative teaching, it is important for teachers to fully understand how they can help their students use their prior knowledge of mathematics and actual learning. The primary focus of the TI model is on students' learning and the creating of mathematical concepts, with achievement as the main goal. This process is individual, with students' prior experience and knowledge forming the basis of learning new mathematical concepts, as illustrated by the theoretical aspects of the TI model (Figure 1). In this analytical study, the focus is specifically on the construction of transformative mathematical instruction called the TI model (RQ1), and logical analytical reasoning about how the TI model's conceptual context can be used in an algebraic-specific context (RQ2). More specifically transformative teaching with a focus on students' learning of algebraic rules and rational equations concerning students' prior knowledge, as well as whole and rational numbers in the framework of a task-oriented learning approach. Examples 1 and 2 below illustrate transformative teaching, guiding students' learning as a transfer within "meaning schemes" from prior knowledge to the generalization of the concept of equations and rational equations, as well as algebraic rules.

## 5.2. Conceptualization of "Meaning Schemes"

*Example 1*

Prior knowledge of inverse operations (addition and subtraction) will activate content and process reflection within the framework of task-oriented teaching of learners' straightforward understanding of a new meaning scheme about negative numbers. A profound understanding of negative numbers can be useful for teaching the generalization of algebraic rules, as well as "two minuses/pluses make a plus and minus plus make a minus" (e.g., Karlsson & Kilborn, 2024b).

*Example 2*

The relationship between rational numbers, ratios, and rational equations has been described by various scholars (e.g., Karlsson & Kilborn, 2024a). Research about the relationship between algebraic concepts can be used in the context of developing students' conceptual knowledge of transformative mathematical instruction: the TI model. Prior knowledge of rational numbers as equivalence classes activates students' straightforward understanding of new concepts of ratio and proportion, and this kind of knowledge is a prerequisite for students' profound comprehension of algebraic concepts and the generalization of rational equations (Karlsson & Kilborn, 2024a).

*Summary*

Examples 1 and 2 illustrate how students can receive diverse types of conceptual knowledge through the organization of teaching using learning schemes and task-oriented teaching strategies, starting with instrumental learning. The same conceptual content is used to design dialogic and self-reflective learning through teaching, with a focus on progression in students' work with actual content via verbalization, reasoning and reflections. The TI model, task-oriented learning schemes and reflection aspects, (see Figure 1), should be included in an appropriate algebra-specific didactic approach, as specifically applied in task-oriented learning.

The benefits of the TI model are more understanding of enrich effective algebra teaching, where student' algebra learning, and knowledge development is crucial. The study describes new theoretical directions in mathematics education with the main contribution being systematic theoretical knowledge of transformative teaching and learning of algebraic concepts.

## 6. FUTURE RESEARCH DIRECTIONS

The theoretical model (TI model) introduced in this study provides opportunities to implement TP and TL theoretical concepts and apply them analytically in future empirical studies. As regards further empirical studies focused on teacher educators' practices related to students' task-oriented learning, the present authors recommend incorporating the theoretical findings from this study in the form of the transformative instruction model (TI model).

The TI model that resulted from this study is very comprehensive and requires further research about transformative teaching and learning mathematics that needs to be expanded both theoretically and empirically. Theoretically, this means constructing more concrete teaching tasks with a focus on mathematical concepts related to the conceptual meaning of learning types and meanings schemes, including reflection and implementation, as well as empirical studies about teachers' transformative professional development and students transformative learning of mathematics, concerning the TI model.

Further empirical studies are needed that focus on the effects of implementing the TI model in a classroom setting, as well as how the TI model influences students' algebra achievement in the long term. This could refine the model based on practical classroom experiences and examine the benefits of the model, as well as its practical implications and challenges.

## 7. DISCUSSION AND CONCLUSION

In this chapter, theoretical aspects of transformative pedagogy (TP) and transformative learning (TL) in a task-oriented teaching context were analyzed to gain a better understanding of what constitutes effective mathematics teaching, with a focus on optimizing students' learning experiences, with examples related to algebraic concepts. The systematic organization and logical track reflection on these theoretical points highlight the significant potential of the study, particularly in the development of a transformative instructional design. The study's analytical findings, such as the transformative mathematical instruction model (TI model) provide advanced research in mathematics education, which combines a psychological perspective about teaching with learning and algebra content. The relationship between the TI model and algebra-specific concepts illustrated by the analytical findings, and particularly by Examples 1 and 2 show how algebraic concepts can interact with students' development in algebra, with a thematic emphasis on instrumental, dialogic and self-reflective task-oriented learning. Researchers implement analytical findings as TI model related to teaching and learning algebra, see Examples 1 and 2. Its led to comprehensively understand the structure of conceptual significance of transformative students' learning and how theoretically, it can be applied in practical teaching. Thus, the results provide an analytical framework for future empirical studies.

The benefits of the TI model are that it illustrates effective mathematics teaching and learning and contributes more knowledge related to arguments 1–5 (see background) especially the interplay between teaching algebra content and students' learning using a transformative approach.

## 8. LIMITATIONS

The current research highlights the limited theoretical and in particularly empirical studies about the transformative teaching of mathematics–specific content that analyzes and illustrates the transfer of concrete learning content from prior knowledge to new knowledge and to applying knowledge, for example, related to algebra or another type of mathematical content. This confirms the positive benefits of the TI model and the results in the form of the implications for algebra in Examples 1 and 2.

Also, a limitation of this study is that while the theoretical foundation for the transformative instruction (TI) model and its application to mathematics teaching has been presented, further empirical research is required to fully validate and explore its effectiveness in diverse educational practical settings. The theoretical approaches to TP and TL, which led to the construction of the TI framework proposed in this study, stimulate discussions on the structural aspects of the task-oriented teaching of algebra, providing a conceptually grounded model for transformative teaching and learning mathematics. The study offers a conceptual framework and examples (Examples 1 and 2) of how the TI model can be applied, particularly in an algebraic context, although it does not provide extensive empirical data or case studies to confirm the validity of the model on teaching practices or student learning outcomes. In addition, the generalizability of the TI model to other areas of mathematics education or to different students' populations remains to be tested in future research.

## 9. IMPLEMENTATION

The implementation of the TI model as designed for teaching and learning took place autumn 2024 in the upper secondary schools of Huddinge municipality in Stockholm with a focus on different types of learning and the transition from students' arithmetic to algebraic knowledge.

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