Chapter # 10

ASSESSMENT OF NUMERICAL COGNITION IN BRAZILIAN PRESCHOOLERS
Elaboration and Study Pilot

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ABSTRACT
During the preschool years, the main difficulty that can predict the risk of mathematical difficulties is the delay in the ability to learn quantities associated with number words and Arabic numerals. Objective: to assess numerical cognition for Brazilian preschoolers aged 2 years to 5 years and 11 months old. Methods: Based on a literature review, mathematical skills were selected to evaluate numerical cognition. A pilot study was conducted to verify the applicability of the developed instrument in 74 Brazilian preschoolers (44 boys) from public municipal early childhood education in the city of São Paulo, Brazil, distributed in the following age groups: Group I: 15 preschoolers, aged 2 years to 2 years and 11 months old; Group II: 18 preschoolers, aged between 3 years and 3 years and 11 months old; Group III: 19 preschoolers, aged between 4 years and 4 years to 11 months old; and Group IV: 22 preschoolers, aged between 5 years and 5 years to 11 months old. Developing an instrument to assess numerical cognition in preschoolers can contribute to developing logical-mathematical reasoning in these children, helping teachers develop activities and games that teach basic numerical, contributing to future gains in mathematics learning.

Keywords: numerosity, mathematical skills, dyscalculia, early childhood, early childhood education.

1. INTRODUCTION

Few studies in the literature investigate mathematical development in children in early childhood (Gersten, Jordan, & Flojo, 2005). In Brazil, this literature is still non-existent. Therefore, the present study proposes to develop an instrument to assess numerical cognition for preschoolers aged between 2 and 5 based on theoretical foundations. Understanding the process of learning mathematics, its difficulties and, consequently, school failure, is a major challenge for all professionals involved in the field of education. The main concern of this work was to promote a better understanding of the difficulties in learning mathematics and the mental processes involved in this learning process in the early years, as well as a clearer and more descriptive diagnosis of the problems in this learning process.

Children's numerical and spatial skills play a critical role in mathematical development, so it is essential to understand the potential impact that age can have on these skills and their relationships with learning (Moeller et al., 2012), as they can be helpful for the development of educational interventions (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007).

Braeuning, Ribner, Moeller, and Blair (2020) state that basic numerical skills acquired before school entry significantly predict later mathematical and educational achievement (Parsons & Bynner, 2005; Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009, Jordan, Glutting, & Ramineni, 2010). These skills are often summarized under the broad early numeracy construct (Aunio & Niemivirta, 2010), an umbrella term that encompasses several skills such as verbal counting, knowledge of numerical symbols, recognizing quantities, discerning numerical patterns, comparing numerical magnitudes, and manipulating quantities (Raghubar & Barnes, 2017).

According to Chu et al. (2019), it is unclear whether children's early foundation for mathematical development rests on an evolved number sense or their understanding of number words and numerals (De Smedt, Noël, Gilmore, & Ansari, 2013; Geary & vanMarle, 2016). Number sense is supported by the approximate number system (ANS) that allows people and other animals to determine the relative quantity of two object collections and perhaps perform basic arithmetic operations on these representations (Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 2000; Geary, Berch, & Mann Koepke, 2015).

2. BACKGROUND

Chu et al. (2019) stated that performance on ANS measures is correlated with concurrent and prospective mathematical performance (Bonny & Lourenco, 2013; Libertus, Feigenson, & Halberda, 2013; Starr, Libertus, & Brannon, 2013), and severe ANS deficits may underlie mathematical learning disabilities in some children (Piazza et al., 2010). Although a relationship between ANS acuity and mathematical performance has been confirmed in several meta-analyses (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2017), the proposal that this is a causal relation is debated (Carey, Shusterman, Haward, & Distefano, 2017; De Smedt et al., 2013; Fuhs & McNeil, 2013; Rousselle & Noël, 2007).

Some authors have developed models that focus on children's initial conceptual understanding of number words and numerals (Chu, vanMarle, & Geary, 2015; vanMarle, Chu, Li, & Geary, 2014) and their later fluency in accessing the corresponding magnitudes (Holloway & Ansari, 2009; Rousselle & Noël, 2007). Children's memorization of number words, the standard counting sequence, and their use of this sequence to enumerate object collections provide the first foothold in symbolic mathematics, but by themselves do not indicate a conceptual understanding of number symbols (Fuson, 1988; Gelman & Gallistel, 1978).

According to Chu et al. (2019), children's first major conceptual insight comes with their understanding of the quantities represented by number words, that is, their cardinal values (Carey, 2004; Le Corre & Carey, 2007; Wynn, 1992). Children's progression on a standard cardinality measure and give-a-number explain this concept's unfolding development (Wynn, 1992). In the give-a-number measure, children are asked to provide n objects from a pile of objects. According to Chu et al. (2019), most 3-year-olds can provide an object when asked to do so but provide random quantities for other number words and are referred to as knowing one. Over the next 1 or 2 years, children will learn the quantities associated with two, three, and four, referred to as knowing two, three, and four, respectively (Le Corre & Carey, 2007). Sometime after this point, children understand that each number word represents a unique quantity and that each successive number is one more than the previous number. Children are then considered to know cardinal principles, although it will take several years to understand that each subsequent number is n + 1 (Cheung, Rubenson, & Barner, 2017; Le Corre & Carey, 2007; Wynn, 1990).

The age at which children achieve cardinal principle status predicts their later numerical and arithmetic skills, controlling for other factors (Chu, vanMarle, Rouder, & Geary, 2018; Geary et al., 2018; Spaepen, Gunderson, Gibson, Goldin-Meadow, & Levine, 2018). Thus, if a fundamental quantitative deficit precedes the emergence of mathematical learning disability, then this deficit likely involves low ANS acuity, delayed understanding of cardinality, or delayed learning of basic numerical skills that precede children's understanding of cardinality. The latter would include knowledge of the counting sequence and the use of this sequence to enumerate items' collections (Fuson, 1988; Gelman & Gallistel, 1978).

For most children, according to Raghubar & Barnes (2017), the acquisition and mastery of early mathematical skills occur spontaneously through activities at home and other experiences in the child's everyday environment (LeFevre et al., 2009). However, this is not the case for all children. For example, children from low-socioeconomic families typically demonstrate less developed numerical knowledge during the preschool and kindergarten years than their middle-income peers (Griffin, Case, & Siegler, 1995; Jordan, Huttenlocher, & Levine, 1994; Starkey, Klein, & Wakeley, 2004), which has been associated with exposure to significantly fewer and less complex everyday numerical activities and experiences (Blevins-Knabe & Musin-Miller, 1996).

According to Raghubar & Barnes (2017), children with learning difficulties related to specific neurodevelopmental disorders or other developmental factors struggle to acquire informal mathematical knowledge. Consequently, children with low numerical skills at the beginning of early childhood education, regardless of the source of this lack of mathematical knowledge, are at high risk of low performance in mathematics in the early years of elementary school (Jordan et al., 2009).

Based on the above, this study hypothesizes that developing an instrument to assess numerical cognition in preschoolers can contribute to the identification of possible difficulties. This study aimed to develop an instrument to evaluate numerical cognition for preschoolers for children aged 2 to 5 years and 11 months old and apply it in a pilot study.

3. METHOD

This study was approved by the Research Ethics Committee of the Faculty of Philosophy and Sciences of the São Paulo State University "Júlio de Mesquita Filho" - FFC/UNESP - Marília-SP, under number 6,138,570 and was divided into phase 1 – Bibliographic survey and preparation of the assessment of numerical cognition for preschoolers and phase 2 – Pilot study.

The guardians who agreed to the assessment of the child/adolescent read and signed the Free and Informed Consent Form. All children included in the study signed the Assent Form.

3.1. Phase 1: Development of the Instrument

Based on the national and international bibliographic survey on numerical cognition assessment skills for the age group of 2 to 5 years and 11 months old, a protocol was developed containing the following abilities: subitizing, knowledge of numerals, counting, one-to-one correspondence, perception of magnitude, cardinality, comparison, measurement and approximation and estimation (the latter only for the age group of 4 years and 11 months to 5 years and 11 months old). In addition to consulting the national and international literature, the activities described in the National Common Curricular Base (BNCC) in the Early Childhood Education division (BRASIL, 2017) were also consulted so that the assessment activities developed in this phase of the study are aligned with the guidelines of this National Literacy Plan.

The bibliographic survey was carried out using the following databases: Scielo Scopus, PubMed, and Web of Science to search for studies on numerical cognition assessment instruments in preschoolers. The bibliographic survey was conducted at the Investigation Learning Disabilities Laboratory (LIDA) of the Speech Therapy Department of the Faculty of Philosophy and Sciences of the São Paulo State University "Júlio de Mesquita Filho- FFC/UNESP- Marília- SP."

Table 1 presents the research results on numerical knowledge in Early Childhood Education, adapted for developing the numerical cognition assessment instrument for preschoolers.

Table 1.
Description of intervention programs with preschoolers described in the literature.

Authors	Year	Objectives	Skills worked on	Age Group
Geary, D. C., van Marle, K., Chu, F. W., Rouder, J., Hoard, M. K., & Nugent, L.	2018	Identify preschool quantitative skills that predict children's knowledge of the school number system.	Identifying the age at which children first understand cardinality, early quantitative skills, number recognition and counting list knowledge, ordinal comparison, and discrete quantity discrimination	3 years and 10 months to 6 years and 9 months old
Sarnecka, B. W., & Lee, M. D.	2009	Analyze the performance pattern of errors when the child is asked for a certain number of objects.	Intransitive counting (reciting the list of number words up to "ten") and transitive counting of arrays of 5 and 10 objects. Giving a requested number of objects	2 years to 4 years old
Sarnecka, B. W., & Carey, S.	2008	Explore children's understanding of how the direction and unit of numerical change are represented by moving forward or backward through the list of numerals	Determine which numerals the child knows the exact meanings of. Measure the child's knowledge of the numeral up to "ten." Measure the child's ability to mark objects in one-to-one correspondence with the list of numbers. Investigate the child's responses to a question about how many follow a standard counting by an adult.	2 years and 10 months to 4 years and 3 months old

Authors	Year	Objectives	Skills worked on	Age Group
Le Corre, M., & Carey, S.	2007	Analyze children's performance in a verbal numerical estimation task.	Give a requested number of objects Counting Estimating	2 years to 5 years old
Jordan, N. C., Kaplan, D., Nabors Oláh, L., & Locuniak, M. N.	2006	Longitudinal investigation of children at risk of difficulties in mathematics for the development of number sense in Early Childhood Education	Measures of number sense: Counting skills, number recognition, number knowledge, nonverbal calculation, narrative problems, number combinations, estimation, and number patterns.	5 years old

3.2. Phase 2: Pilot Study

The study included 74 Brazilian preschoolers (44 boys and 30 girls) from public municipal early childhood education in the city of São Paulo, distributed in the following age groups: Group I (GI): composed of 15 preschoolers aged 2 years to 2 years and 11 months old; Group II (GII): composed of 18 preschoolers, aged 3 years to 3 years and 11 months old; (GIII): composed of 19 preschoolers, aged 4 years to 4 years to 11 months old; and (GIV): composed of 22 preschoolers, aged 5 years to 5 years to 11 months old.

All preschoolers underwent the assessment of numerical cognition for preschoolers. The numerical tasks were designed to be applied as games to maintain attention. Short breaks were provided between tasks, as necessary. The researcher assessed the participants individually in a room within the school environment.

Non-parametric statistical tests were used to analyze the results since the normality of the primary outcome quantitative variables was tested using the Kolmogorov-Smirnov test (N<100), and it was concluded that there is no guaranteed normal distribution. A significant level of 0.05 (5%) was adopted.

4. RESULTS

The analyses began with an Intragroup comparison. For each group, the Obtained and Expected results were compared, starting with the quantitative ones, using the Wilcoxon test.

Table 2.
Intragroup Comparison for Quantitative Factors.

			Average	Median	Standard Deviation	Q1	Q3	IQR	N	IC	P-value
Verbal counting	Group 1	Obtained	2,04	1	2,96	0	2,75	2,75	26	1,14	0,001*
		Expected	5,00	5	0,00	5	5	0	26	- x -	
	Group 2	Obtained	5,12	4	4,56	1	10	9	49	1,28	<0,001*
		Expected	10,00	10	0,00	10	10	0	49	- x -	
	Group 3	Obtained	13,20	10	13,48	5	17,5	12,5	71	3,13	0,119
		Expected	10,00	10	0,00	10	10	0	71	- x -	
	Group 4	Obtained	26,22	20	19,51	10	38	28	73	4,48	<0,001*
		Expected	10,00	10	0,00	10	10	0	73	- x -	
Sequence counting	Group 1	Obtained	2,31	0	3,22	0	4,5	4,5	26	1,24	0,001*
		Expected	5,00	5	0,00	5	5	0	26	- x -	
	Group 2	Obtained	4,53	3	4,01	1	10	9	49	1,12	<0,001*
		Expected	10,00	10	0,00	10	10	0	49	- x -	
	Group 3	Obtained	7,65	10	3,25	5	10	5	71	0,76	<0,001*
		Expected	10,00	10	0,00	10	10	0	71	- x -	
	Group 4	Obtained	9,56	10	1,65	10	10	0	73	0,38	0,042*
		Expected	10,00	10	0,00	10	10	0	73	- x -	
One-to-one correspondence tokens	Group 1	Obtained	3,50	1,5	3,67	1	5	4	26	1,41	<0,001*
		Expected	10,00	10	0,00	10	10	0	26	- x -	
	Group 2	Obtained	5,78	5	4,06	2	10	8	49	1,14	<0,001*
		Expected	10,00	10	0,00	10	10	0	49	- x -	
	Group 3	Obtained	8,15	10	3,05	5	10	5	71	0,71	<0,001*
		Expected	10,00	10	0,00	10	10	0	71	- x -	
	Group 4	Obtained	9,75	10	1,21	10	10	0	73	0,28	0,109
		Expected	10,00	10	0,00	10	10	0	73	- x -	

It was found that for Verbal Count, there is a difference between Obtained and Expected for Groups 1, 2, and 4 (P-value of 0.001, <0.001, and <0.001, respectively), and in Group 3, we have no statistical difference with an average of 13.20 for Obtained against 10.00 for Expected (p-value = 0.119).

Sequence Count is significant in the four groups, where the average for Obtained was consistently below Expected, as in Group 1, with an average of 2.31 for Obtained and 5.00 for Expected (p-value = 0.001).

Finally, in "One-to-one correspondence of tokens," it was verified that the significance between Obtained and Expected occurred again in groups 1, 2, and 3. In Group 1, for example, the average for Obtained was 3.53 against 10.0 for Expected (P-value = 0.002).

The obtained and expected results were compared, but the Chi-Square test determined the distribution of qualitative factors (percentages or prevalences).

Table 3.
Intragroup Comparison for Distribution of Qualitative Factors.

			Expected		**Obtained**		**P-Value**
			N	%	N	%	
Cardinality 5	Group 1	Correct	0	0,0%	3	11,5%	0,074
		Incorrect	26	100,0%	23	88,5%	
	Group 2	Correct	49	100,0%	15	30,6%	<0,001*
		Incorrect	0	0,0%	34	69,4%	
	Group 3	Correct	71	100,0%	50	70,4%	<0,001*
		Incorrect	0	0,0%	21	29,6%	
	Group 4	Correct	73	100,0%	61	83,6%	<0,001*
		Incorrect	0	0,0%	12	16,4%	
Cardinality 7	Group 1	Correct	0	0,0%	1	3,8%	0,313
		Incorrect	26	100,0%	25	96,2%	
	Group 2	Correct	49	100,0%	6	12,2%	<0,001*
		Incorrect	0	0,0%	43	87,8%	
	Group 3	Correct	71	100,0%	37	52,1%	<0,001*
		Incorrect	0	0,0%	34	47,9%	
	Group 4	Correct	73	100,0%	61	83,6%	<0,001*
		Incorrect	0	0,0%	12	16,4%	
Cardinality enumeration counting 5	Group 1	Correct	26	100,0%	10	38,5%	<0,001*
		Incorrect	0	0,0%	16	61,5%	
	Group 2	Correct	49	100,0%	27	55,1%	<0,001*
		Incorrect	0	0,0%	22	44,9%	
	Group 3	Correct	71	100,0%	61	85,9%	<0,001*
		Incorrect	0	0,0%	10	14,1%	
	Group 4	Correct	73	100,0%	69	94,5%	0,043*
		Incorrect	0	0,0%	4	5,5%	
Cardinality enumeration counting 7	Group 1	Correct	0	0,0%	4	15,4%	0,037*
		Incorrect	26	100,0%	22	84,6%	
	Group 2	Correct	49	100,0%	14	28,6%	<0,001*
		Incorrect	0	0,0%	35	71,4%	
	Group 3	Correct	71	100,0%	48	67,6%	<0,001*
		Incorrect	0	0,0%	23	32,4%	
	Group 4	Correct	73	100,0%	66	90,4%	0,007*
		Incorrect	0	0,0%	7	9,6%	
Cardinality colored tokens	Group 1	Knowledgeable	0	0,0%	0	0,0%	- x -
		Not Knowledgeable	26	100,0%	26	100,0%	
	Group 2	Knowledgeable	49	100,0%	7	14,3%	<0,001*
		Not Knowledgeable	0	0,0%	42	85,7%	
	Group 3	Knowledgeable	71	100,0%	38	53,5%	<0,001*
		Not Knowledgeable	0	0,0%	33	46,5%	

	Group 4	Knowledgeable	73	100,0%	64	87,7%	0,002*
		Not Knowledgeable	0	0,0%	9	12,3%	
Cardinality last word	Group 1	Knowledgeable	0	0,0%	1	3,8%	0,313
		Not Knowledgeable	26	100,0%	25	96,2%	
	Group 2	Knowledgeable	49	100,0%	9	18,4%	<0,001*
		Not Knowledgeable	0	0,0%	40	81,6%	
	Group 3	Knowledgeable	71	100,0%	38	53,5%	<0,001*
		Not Knowledgeable	0	0,0%	33	46,5%	
	Group 4	Knowledgeable	73	100,0%	60	82,2%	<0,001*
		Not Knowledgeable	0	0,0%	13	17,8%	
Data comparison	Group 1	Correct	26	100,0%	15	57,7%	<0,001*
		Incorrect	0	0,0%	11	42,3%	
	Group 2	Correct	49	100,0%	33	67,3%	<0,001*
		Incorrect	0	0,0%	16	32,7%	
	Group 3	Correct	71	100,0%	63	88,7%	0,004*
		Incorrect	0	0,0%	8	11,3%	
	Group 4	Correct	73	100,0%	72	98,6%	0,316
		Incorrect	0	0,0%	1	1,4%	
Domino comparison	Group 1	Correct	26	100,0%	16	61,5%	<0,001*
		Incorrect	0	0,0%	10	38,5%	
	Group 2	Correct	49	100,0%	37	75,5%	<0,001*
		Incorrect	0	0,0%	12	24,5%	
	Group 3	Correct	71	100,0%	63	88,7%	0,004*
		Incorrect	0	0,0%	8	11,3%	
	Group 4	Correct	73	100,0%	72	98,6%	0,316
		Incorrect	0	0,0%	1	1,4%	
Comparison of different objects	Group 1	Correct	26	100,0%	2	7,7%	<0,001*
		Incorrect	0	0,0%	24	92,3%	
	Group 2	Correct	49	100,0%	22	44,9%	<0,001*
		Incorrect	0	0,0%	27	55,1%	
	Group 3	Correct	71	100,0%	47	66,2%	<0,001*
		Incorrect	0	0,0%	24	33,8%	
	Group 4	Correct	73	100,0%	60	82,2%	<0,001*
		Incorrect	0	0,0%	13	17,8%	
Comparison of similar objects	Group 1	Correct	26	100,0%	8	30,8%	<0,001*
		Incorrect	0	0,0%	18	69,2%	
	Group 2	Correct	49	100,0%	30	61,2%	<0,001*
		Incorrect	0	0,0%	19	38,8%	
	Group 3	Correct	71	100,0%	49	69,0%	<0,001*
		Incorrect	0	0,0%	22	31,0%	
	Group 4	Correct	73	100,0%	64	87,7%	0,002*
		Incorrect	0	0,0%	9	12,3%	

One-to-one correspondence between each student and their book	Group 1	Correct	26	100,0%	26	100,0%	- x -
		Incorrect	0	0,0%	0	0,0%	
	Group 2	Correct	49	100,0%	48	98,0%	0,315
		Incorrect	0	0,0%	1	2,0%	
	Group 3	Correct	71	100,0%	71	100,0%	- x -
		Incorrect	0	0,0%	0	0,0%	
	Group 4	Correct	73	100,0%	73	100,0%	- x -
		Incorrect	0	0,0%	0	0,0%	
Domino measurement	Group 1	Correct	26	100,0%	20	76,9%	0,009*
		Incorrect	0	0,0%	6	23,1%	
	Group 2	Correct	49	100,0%	35	71,4%	<0,001*
		Incorrect	0	0,0%	14	28,6%	
	Group 3	Correct	71	100,0%	67	94,4%	0,042*
		Incorrect	0	0,0%	4	5,6%	
	Group 4	Correct	73	100,0%	71	97,3%	0,154
		Incorrect	0	0,0%	2	2,7%	
Elephant/Dog Measurement	Group 1	Correct	26	100,0%	22	84,6%	0,037*
		Incorrect	0	0,0%	4	15,4%	
	Group 2	Correct	49	100,0%	42	85,7%	0,006*
		Incorrect	0	0,0%	7	14,3%	
	Group 3	Correct	71	100,0%	66	93,0%	0,023*
		Incorrect	0	0,0%	5	7,0%	
	Group 4	Correct	73	100,0%	73	100,0%	- x -
		Incorrect	0	0,0%	0	0,0%	
Ice/fire Measurement	Group 1	Correct	26	100,0%	15	57,7%	<0,001*
		Incorrect	0	0,0%	11	42,3%	
	Group 2	Correct	49	100,0%	36	73,5%	<0,001*
		Incorrect	0	0,0%	13	26,5%	
	Group 3	Correct	71	100,0%	63	88,7%	0,004*
		Incorrect	0	0,0%	8	11,3%	
	Group 4	Correct	73	100,0%	72	98,6%	0,316
		Incorrect	0	0,0%	1	1,4%	
Magnitude perception 1 by 2	Group 1	Correct	26	100,0%	13	50,0%	<0,001*
		Incorrect	0	0,0%	13	50,0%	
	Group 2	Correct	49	100,0%	38	77,6%	<0,001*
		Incorrect	0	0,0%	11	22,4%	
	Group 3	Correct	71	100,0%	58	81,7%	<0,001*
		Incorrect	0	0,0%	13	18,3%	
	Group 4	Correct	73	100,0%	71	97,3%	0,154
		Incorrect	0	0,0%	2	2,7%	
Magnitude perception 2 by 3	Group 1	Correct	26	100,0%	10	38,5%	<0,001*
		Incorrect	0	0,0%	16	61,5%	
	Group 2	Correct	49	100,0%	31	63,3%	<0,001*
		Incorrect	0	0,0%	18	36,7%	
	Group 3	Correct	71	100,0%	59	83,1%	<0,001*

		Incorrect	0	0,0%	12	16,9%	
	Group 4	Correct	73	100,0%	69	94,5%	0,043*
		Incorrect	0	0,0%	4	5,5%	
Magnitude perception 3 by 4	Group 1	Correct	26	100,0%	10	38,5%	<0,001*
		Incorrect	0	0,0%	16	61,5%	
	Group 2	Correct	49	100,0%	25	51,0%	<0,001*
		Incorrect	0	0,0%	24	49,0%	
	Group 3	Correct	71	100,0%	54	76,1%	<0,001*
		Incorrect	0	0,0%	17	23,9%	
	Group 4	Correct	73	100,0%	64	87,7%	0,002*
		Incorrect	0	0,0%	9	12,3%	
Magnitude perception 4 by 5	Group 1	Correct	26	100,0%	14	53,8%	<0,001*
		Incorrect	0	0,0%	12	46,2%	
	Group 2	Correct	49	100,0%	29	59,2%	<0,001*
		Incorrect	0	0,0%	20	40,8%	
	Group 3	Correct	71	100,0%	55	77,5%	<0,001*
		Incorrect	0	0,0%	16	22,5%	
	Group 4	Correct	73	100,0%	56	76,7%	<0,001*
		Incorrect	0	0,0%	17	23,3%	
Magnitude perception 5 by 6	Group 1	Correct	26	100,0%	10	38,5%	<0,001*
		Incorrect	0	0,0%	16	61,5%	
	Group 2	Correct	49	100,0%	22	44,9%	<0,001*
		Incorrect	0	0,0%	27	55,1%	
	Group 3	Correct	71	100,0%	45	63,4%	<0,001*
		Incorrect	0	0,0%	26	36,6%	
	Group 4	Correct	73	100,0%	55	75,3%	<0,001*
		Incorrect	0	0,0%	18	24,7%	
Magnitude perception 6 by 7	Group 1	Correct	26	100,0%	9	34,6%	<0,001*
		Incorrect	0	0,0%	17	65,4%	
	Group 2	Correct	49	100,0%	25	51,0%	<0,001*
		Incorrect	0	0,0%	24	49,0%	
	Group 3	Correct	71	100,0%	52	73,2%	<0,001*
		Incorrect	0	0,0%	19	26,8%	
	Group 4	Correct	73	100,0%	59	80,8%	<0,001*
		Incorrect	0	0,0%	14	19,2%	
Data subitizing	Group 1	Correct	26	100,0%	5	19,2%	<0,001*
		Incorrect	0	0,0%	21	80,8%	
	Group 2	Correct	49	100,0%	26	53,1%	<0,001*
		Incorrect	0	0,0%	23	46,9%	
	Group 3	Correct	71	100,0%	55	77,5%	<0,001*
		Incorrect	0	0,0%	16	22,5%	
	Group 4	Correct	73	100,0%	68	93,2%	0,023*
		Incorrect	0	0,0%	5	6,8%	
Domino subitizing	Group 1	Correct	26	100,0%	9	34,6%	<0,001*
		Incorrect	0	0,0%	17	65,4%	

	Group 2	Correct	49	100,0%	27	55,1%	<0,001*
		Incorrect	0	0,0%	22	44,9%	
	Group 3	Correct	71	100,0%	53	74,6%	<0,001*
		Incorrect	0	0,0%	18	25,4%	
	Group 4	Correct	73	100,0%	69	94,5%	0,043*
		Incorrect	0	0,0%	4	5,5%	
Subitizing sticks	Group 1	Correct	26	100,0%	7	26,9%	<0,001*
		Incorrect	0	0,0%	19	73,1%	
	Group 2	Correct	49	100,0%	29	59,2%	<0,001*
		Incorrect	0	0,0%	20	40,8%	
	Group 3	Correct	71	100,0%	57	80,3%	<0,001*
		Incorrect	0	0,0%	14	19,7%	
	Group 4	Correct	73	100,0%	65	89,0%	0,004*
		Incorrect	0	0,0%	8	11,0%	
One-to-one correspondence of ten objects	Group 1	Correct with Help	0	0,0%	7	26,9%	0,004*
		Correct without Help	26	100,0%	3	11,5%	<0,001*
		Incorrect	0	0,0%	16	61,5%	<0,001*
	Group 2	Correct with Help	0	0,0%	9	18,4%	0,002*
		Correct without Help	49	100,0%	16	32,7%	<0,001*
		Incorrect	0	0,0%	24	49,0%	<0,001*
	Group 3	Correct with Help	0	0,0%	16	22,5%	<0,001*
		Correct without Help	71	100,0%	41	57,7%	<0,001*
		Incorrect	0	0,0%	14	19,7%	<0,001*
	Group 4	Correct with Help	0	0,0%	3	4,1%	0,080
		Correct without Help	73	100,0%	68	93,2%	0,023*
		Incorrect	0	0,0%	2	2,7%	0,154
Numeral knowledge	Group 1	Not Numeric	0	0,0%	7	26,9%	0,004*
		One	0	0,0%	4	15,4%	0,037*
		Two	0	0,0%	9	34,6%	<0,001*
		Three	26	100,0%	4	15,4%	<0,001*
		Four	0	0,0%	2	7,7%	0,149
		>4	0	0,0%	0	0,0%	- x -
	Group 2	Not Numeric	0	0,0%	5	10,2%	0,022*
		One	0	0,0%	6	12,2%	0,011*
		Two	0	0,0%	11	22,4%	<0,001*
		Three	0	0,0%	13	26,5%	<0,001*
		Four	49	100,0%	9	18,4%	<0,001*
		>4	0	0,0%	5	10,2%	0,022*
	Group 3	Not Numeric	0	0,0%	0	0,0%	- x -
		One	0	0,0%	0	0,0%	- x -
		Two	0	0,0%	7	9,9%	0,007*

		Three	0	0,0%	10	14,1%	0,001*
		Four	0	0,0%	18	25,4%	<0,001*
		>4	71	100,0%	36	50,7%	<0,001*
	Group 4	Not Numeric	0	0,0%	0	0,0%	- x -
		One	0	0,0%	1	1,4%	0,316
		Two	0	0,0%	3	4,1%	0,080
		Three	0	0,0%	2	2,7%	0,154
		Four	0	0,0%	6	8,2%	0,012*
		>4	73	100,0%	61	83,6%	<0,001*
Numeral knowledge (Groups 4)	Dinosaurs	Correct	73	100,0%	40	54,8%	<0,001*
		Incorrect	0	0,0%	33	45,2%	
	Tokens <10	Correct	73	100,0%	44	60,3%	<0,001*
		Incorrect	0	0,0%	29	39,7%	
	Tokens >30	Correct	73	100,0%	44	60,3%	<0,001*
		Incorrect	0	0,0%	29	39,7%	
	Points 15	Correct	73	100,0%	14	19,2%	<0,001*
		Incorrect	0	0,0%	59	80,8%	
	Points 25	Correct	73	100,0%	18	24,7%	<0,001*
		Incorrect	0	0,0%	55	75,3%	
	Points 3	Correct	73	100,0%	70	95,9%	0,080
		Incorrect	0	0,0%	3	4,1%	
	Points 35	Correct	73	100,0%	12	16,4%	<0,001*
		Incorrect	0	0,0%	61	83,6%	
	Points 8	Correct	73	100,0%	46	63,0%	<0,001*
		Incorrect	0	0,0%	27	37,0%	

It was concluded that there is a statistical difference between the Obtained and Expected for several factors in all groups. However, in addition to the significant results, we can also talk about the non-significant ones, as they show that the group had an Obtained result considered statistically equal to the Expected one. An example is the result in "Cardinality 7," where in Group 1, the Correct rate was 3.8%, statistically equal to the 0.0% of the Expected one (p-value = 0.313).

Another example to mention is the result of the "Domino comparison," where it was observed that in Group 4, the rate between Obtained and Expected are statistically equal, as for the correct one, which was 98.6% in the Obtained against 100% in the Expected one (p-value = 0.316). For this same parameter in Group 3, the Correct rate was 88.7%, statistically different from the 100% of the Expected one (p-value = 0.004).

In addition to the intragroup analyses, intergroup results were performed, where the four groups were compared for the results obtained, starting with the quantitative factors using the Kruskal-Wallis test.

Table 4.
Intergroup Comparison for Quantitative Factors.

		Average	**Median**	**Standard Deviation**	**Q1**	**Q3**	**IQR**	**N**	**IC**	**P-Value**
Verbal counting	Group 1	2,04	1	2,96	0	2,75	2,75	26	1,14	<0,001*
	Group 2	5,12	4	4,56	1	10	9	49	1,28	
	Group 3	13,20	10	13,48	5	17,5	12,5	71	3,13	
	Group 4	26,22	20	19,51	10	38	28	73	4,48	
Sequence count	Group 1	2,31	0	3,22	0	4,5	4,5	26	1,24	<0,001*
	Group 2	4,53	3	4,01	1	10	9	49	1,12	
	Group 3	7,65	10	3,25	5	10	5	71	0,76	
	Group 4	9,56	10	1,65	10	10	0	73	0,38	
One-to-one correspondence tokens	Group 1	3,50	1,5	3,67	1	5	4	26	1,41	<0,001*
	Group 2	5,78	5	4,06	2	10	8	49	1,14	
	Group 3	8,15	10	3,05	5	10	5	71	0,71	
	Group 4	9,75	10	1,21	10	10	0	73	0,28	

Table 5.
P-values of the post-hoc groups in the quantitative factors.

		Group 1	**Group 2**	**Group 3**
Verbal counting	Group 2	0,002*		
	Group 3	<0,001*	<0,001*	
	Group 4	<0,001*	<0,001*	<0,001*
Sequence count	Group 2	0,008*		
	Group 3	<0,001*	<0,001*	
	Group 4	<0,001*	<0,001*	<0,001*
One-to-one correspondence tokens	Group 2	0,016*		
	Group 3	<0,001*	0,001*	
	Group 4	<0,001*	<0,001*	<0,001*

Since there were four groups, all the groups were first compared together (Kruskal-Wallis test in Table 4), and it was concluded from the p-values in Table 4 that there was a statistical difference between the groups in the three factors analyzed. Thus, it was necessary to specify the comparison between the groups in pairs to determine precisely which groups the difference occurs between. Therefore, only these p-values were presented in Table 5 (using the Mann-Whitney test), where it is enough to cross the row with the column to find the p-value between the necessary groups. In general, there is a difference between all the groups in the three parameters.

For example, the result for Verbal Counting is an average of 2.04 in G1, 5.12 in G2, 13.20 in G3, and 26.22 in G4. Analyzing the p-values in the post-hoc table (Table 5), we concluded that there is a difference between all the groups.

Finally, the groups were compared for the distribution of qualitative factors by analyzing the percentages using the Chi-Square test.

Table 6.
Intergroup Comparison for the Distribution of Qualitative Factors.

		Group 1		**Group 2**		**Group 3**		**Group 4**		**P-Value**
		N	%	N	%	N	%	N	%	
Cardinality 5	Correct	3	11,5%	15	30,6%	50	70,4%	61	83,6%	<0,001*
	Incorrect	23	88,5%	34	69,4%	21	29,6%	12	16,4%	
Cardinality 7	Correct	1	3,8%	6	12,2%	37	52,1%	61	83,6%	<0,001*
	Incorrect	25	96,2%	43	87,8%	34	47,9%	12	16,4%	
Cardinality enumeration counting 5	Correct	10	38,5%	27	55,1%	61	85,9%	69	94,5%	<0,001*
	Incorrect	16	61,5%	22	44,9%	10	14,1%	4	5,5%	
Cardinality enumeration counting 7	Correct	4	15,4%	14	28,6%	48	67,6%	66	90,4%	<0,001*
	Incorrect	22	84,6%	35	71,4%	23	32,4%	7	9,6%	
Cardinality colored tokens	Knowledgeable	0	0,0%	7	14,3%	38	53,5%	64	87,7%	<0,001*
	Not Knowledgeable	26	100,0%	42	85,7%	33	46,5%	9	12,3%	
Cardinality last word	Knowledgeable	1	3,8%	9	18,4%	38	53,5%	60	82,2%	<0,001*
	Not Knowledgeable	25	96,2%	40	81,6%	33	46,5%	13	17,8%	
Data comparison	Correct	15	57,7%	33	67,3%	63	88,7%	72	98,6%	<0,001*
	Incorrect	11	42,3%	16	32,7%	8	11,3%	1	1,4%	
Domino comparison	Correct	16	61,5%	37	75,5%	63	88,7%	72	98,6%	0,001*
	Incorrect	10	38,5%	12	24,5%	8	11,3%	1	1,4%	
Comparison of different objects	Correct	2	7,7%	22	44,9%	47	66,2%	60	82,2%	<0,001*
	Incorrect	24	92,3%	27	55,1%	24	33,8%	13	17,8%	
Comparison of similar objects	Correct	8	30,8%	30	61,2%	49	69,0%	64	87,7%	<0,001*
	Incorrect	18	69,2%	19	38,8%	22	31,0%	9	12,3%	
One-to-one correspondence between each student and their book	Correct	26	100,0%	48	98,0%	71	100,0%	73	100,0%	0,369
	Incorrect	0	0,0%	1	2,0%	0	0,0%	0	0,0%	
Domino measurement	Correct	20	76,9%	35	71,4%	67	94,4%	71	97,3%	<0,001*
	Incorrect	6	23,1%	14	28,6%	4	5,6%	2	2,7%	
Elephant/Dog Measurement	Correct	22	84,6%	42	85,7%	66	93,0%	73	100,0%	0,110
	Incorrect	4	15,4%	7	14,3%	5	7,0%	0	0,0%	
Ice/fire measurement	Correct	15	57,7%	36	73,5%	63	88,7%	72	98,6%	<0,001*
	Incorrect	11	42,3%	13	26,5%	8	11,3%	1	1,4%	
Magnitude perception 1 by 2	Correct	13	50,0%	38	77,6%	58	81,7%	71	97,3%	<0,001*
	Incorrect	13	50,0%	11	22,4%	13	18,3%	2	2,7%	

Magnitude perception 2 by 3	Correct	10	38,5%	31	63,3%	59	83,1%	69	94,5%	<0,001*
	Incorrect	16	61,5%	18	36,7%	12	16,9%	4	5,5%	
Magnitude perception 3 by 4	Correct	10	38,5%	25	51,0%	54	76,1%	64	87,7%	<0,001*
	Incorrect	16	61,5%	24	49,0%	17	23,9%	9	12,3%	
Magnitude perception 4 by 5	Correct	14	53,8%	29	59,2%	55	77,5%	56	76,7%	0,045*
	Incorrect	12	46,2%	20	40,8%	16	22,5%	17	23,3%	
Magnitude perception 5 by 6	Correct	10	38,5%	22	44,9%	45	63,4%	55	75,3%	0,016*
	Incorrect	16	61,5%	27	55,1%	26	36,6%	18	24,7%	
Magnitude perception 6 by 7	Correct	9	34,6%	25	51,0%	52	73,2%	59	80,8%	<0,001*
	Incorrect	17	65,4%	24	49,0%	19	26,8%	14	19,2%	
Data subitizing	Correct	5	19,2%	26	53,1%	55	77,5%	68	93,2%	<0,001*
	Incorrect	21	80,8%	23	46,9%	16	22,5%	5	6,8%	
Domino subitizing	Correct	9	34,6%	27	55,1%	53	74,6%	69	94,5%	<0,001*
	Incorrect	17	65,4%	22	44,9%	18	25,4%	4	5,5%	
Subitizing sticks	Correct	7	26,9%	29	59,2%	57	80,3%	65	89,0%	<0,001*
	Incorrect	19	73,1%	20	40,8%	14	19,7%	8	11,0%	

In this study, we found that preschoolers in the 4 groups performed similarly in object comparison, measurement, and magnitude perception skills, showing that these skills did not differentiate the groups.

5. FUTURE RESEARCH DIRECTIONS

Based on the results of this study, a study of the applicability of the assessment procedure developed in phase 1 will be conducted to establish reference standards for the procedure, which will be used to identify Brazilian preschool children at risk for mathematical problems early on.

According to Gersten et al. (2005), the development of number sense can be improved by formal or informal instruction before school entry. At least in 5- and 6-year-old children, the two components of number sense (counting/simple calculation and magnitude sense/use of mental number line) are not well linked. Creating such relationships from the beginning may be critical for developing proficiency in mathematics. Children who have not acquired these relationships probably require an intervention that builds them. Therefore, more research investigating children's numerical knowledge in Early Childhood Education for early identification and intervention is welcome.

6. CONCLUSION/DISCUSSION

The results of this study showed that, based on the literature, an instrument to assess numerical cognitive skills for Brazilian preschoolers aged 2 to 5 years old could be developed.

The ability to subitize small quantities, discern numerical patterns, compare numerical magnitudes and estimate quantities, and count and perform simple numerical transformations are critical elements of number sense in young children (Berch, 2005). Most children develop number sense before formal instruction (Ginsburg & Golbeck, 2004).

One-to-one correspondence appears around two years old, independently of learning the sequence of counting words. According to Butterworth (2005), children can give each person a piece of candy at two years old. In our study, in the one-to-one correspondence activity, children from all groups could give each child a book when asked to do so. However, within the same activity, in counting up to 10 tokens and up to 10 dinosaurs, these activities were more assertive from group 2 onwards. The pilot study showed that verbal counting, sequential counting, and one-to-one correspondence were sensitive skills in characterizing preschoolers' performance according to age group.

According to Geary et al. (2018), children's initial steps in symbolic mathematics include learning the sequence of number words (counting list), recognizing numerals, and counting objects (Geary & vanMarle, 2016; Gelman & Gallistel, 1978; Wynn, 1990). However, this can occur without conceptualizing the symbol or procedure. These activities gain meaning from children's understanding of the cardinal value of number words. Still, the path to this knowledge is long and complex, as illustrated by the task of cardinality-colored tokens (Sarnecka & Carey, 2008).

Children were asked to provide an x number of tokens from a stack of tokens to the researcher. "One" knower provided one token when asked to do so, but random quantities for other number words. "One" knower status is usually achieved between 2 and 3 years old, as was found in our research. It takes children 3 to 6 months to learn the quantities associated with each successive number word up to four. Sometime later, children conceptualize that each number word represents a unique quantity and that successive numbers in their counting list are one more than the previous number (Le Corre & Carey, 2007).

The results of this study are consistent with Jordan, Glutting, and Ramineni (2008), who suggested that interrelated skills involving numbers and operations develop between the ages of 3 and 6, such as subitizing, identifying quantities without counting, counting items in a set of at least five, knowing that the final word of the count indicates how many are in the set.

Developing an instrument to assess numerical cognition in preschoolers can help develop logical-mathematical reasoning in these children, assisting teachers in developing activities and games that teach basic numerical, spatial, geometric, measurement, and statistical concepts, contributing to future gains in mathematics learning.

This study showed that a procedure to assess numerical cognitive skills for Brazilian preschoolers could be developed, and it could also indicate which skills differentiated the groups by age group.

The relatively brief numerical cognition assessment protocol is a valid and powerful measure that can predict which children will likely have problems learning mathematics at the beginning of school. According to Jordan et al. (2010), number sense is an intermediate skill that can be achieved through early instruction (Ginsburg, Lee, & Boyd, 2008). Number sense, which involves interrelated concepts of counting, numerical knowledge, and operations, as presented in our battery, promises to guide the development of early intervention programs.

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