Chapter #17

NATURE OF SCIENCE INTERDISCIPLINARY TEACHING AT PRIMARY SCHOOL BASED ON SYMMETRY AND THE SEARCH OF INVARIANTS

Simone Brasili, & Riccardo Piergallini
Mathematics Division, School of Science and Technology, University of Camerino, Italy

ABSTRACT
The project is situated in the field of teaching physics, generally speaking, science education. It aims at studying the interplay between physics and mathematics, introducing an interdisciplinary approach based on the modern concept of symmetry or sameness within change (i.e., invariance). The interdisciplinary methods integrate the process skills common to discovery-based science and problem-based mathematics, typically of Nature of Science (NoS) modelling. We designed a teaching-learning sequence (TLS) and implemented it in an Italian primary school on a sample group (N = 96). Research conforms to pre- and post-test design with a control group. The resulting data were examined through a mixed method. We assess the effectiveness of the TLS by analysing the learning changes in the level of pupils' knowledge of symmetry. The findings show that the pupils were in the regime of competence for grasping the changed view of symmetry. It also provides ideas that the concepts of symmetry and invariance will allow building the architecture of more extensive scientific knowledge. The study of transformations and invariants facilitates the acquisition of cognitive procedures fitting in many domains, modelling and generalization processes. This research contribution brings important suggestions for designs of successive steps in the learning path on symmetry.

Keywords: teaching science, NoS, symmetry, invariants in science education.

1. INTRODUCTION

1.1. Background

Remarkable studies (Pisano, 2011; Boniolo, Budinich, & Trobok, 2005; Fuson, Kalchman, & Bransford, 2005) have been devoted to the problem of teaching together mathematical and science contents. In addition, the language of mathematics makes sense of a wide range of natural phenomena (Wigner, 1967, p. 230). Mathematics is a vehicle of meaning and sense, and the mastery of symbols is essential to move from one level of abstraction to another of a higher degree. One of the most significant examples is the symmetry and the different forms used in the scientific disciplines throughout history.

Despite the fact that mathematical proficiency is highly relevant in and to the research fields, mathematics education is failing to support interdisciplinary and educational connections to physics. In fact, sometimes from students' prospective, mathematics, physics and biology typically appear as separate subjects with few interconnections (Redish & Kuo, 2015).

This compartmentalization generates many theoretical misunderstandings in the process of learning certain important concepts (Meltzer, 2002; Buick, 2007). For instance, students showing adequate computational skills still lack the ability to apply these skills in a meaningful way. Leikin, Berman, and Zaslavsky (2000a) illustrate the difficulties and misconceptions for seven eight-graders students associated with the symmetry transformations. The issue is exposed by Seah & Horne (2019), Ng & Sinclair (2015), Knuchel (2004) in the elementary school. The continuity and ruptures identified in the use of the symmetry concept are analyzed by Chesnais and Munier (2013) at the transition from primary to secondary school in France. There is evidence that students focus their attention more on basic comprehension and memorization of concepts and formulas than on critical thinking. In this sense, a key aspect of educational innovation is to promote creative and flexible frameworks for integrating productive ideas across disciplines. Symmetry is a unifying concept due to its ability to connect a variety of domains (Dreyfus & Eisenberg, 1990, p. 53). The relationship between symmetry and invariance in mathematics and science has been symbiotic, each contributing to the other's development (Klein, 1990; Weyl, 1952, p.135). Indeed, invariance is very relevant in teaching and learning geometry, but it does not receive the attention it deserves in education (Libeskind, Stupel, & Oxman, 2018; Schuster, 1971). The activities described below are designed to achieve this topic in the teaching-learning process at primary school.

1.2. Focus of the study and literature review

Our project aims at contributing to the research in the field of Nature of Science Teaching regarding the interplay between physics and mathematics (Pisano, 2011; Doran 2017; Wigner, 1960) within an interdisciplinary approach based on symmetry and invariance. Symmetry and searching of invariants can be employed in interdisciplinary perspective because of its crosscutting character and its historico-epistemological value (Dirac, 1939; Feynman, 1964-1965; Galileo, 1623). The possibility of widening the fields of application in which to choose the rules to be considered from time to time offers the starting point for reflections on the relationships of equality and more generally of equivalence, on the relativity of the concept of equality, of form as a physical system that is preserved (Rosen & Copié, 1982; Weyl, 1928; Wigner, 1967).

The processes of change or modelling of a rule are themselves subject to a principle of legality[1] that guarantees compatibility and consistency with those already in existence: in physics, this set of binding conditions is represented by the principles of symmetry. The symmetry perceived with the meaning of the invariance of a form undergoing a change becomes a powerful tool of inter-disciplinary knowledge included applied sciences both into history (Pisano, 2011) and in society (Marchis, 2009). According to Darvas (1997, p. 328), symmetry as a mathematical tool is also advanced in a general scientific method. Thus, it can establish principles generally applicable throughout the sciences.

The essential idea is to extend symmetry starting from line symmetry and aesthetic qualities with the search for regularity to more general and interdisciplinary aspect of dynamic principles of transformation (Leikin, Berman, and Zaslavsky, 2000b). In fact, the figures that have characteristics in common to our eye from different points of view somehow refer to the concept of invariant to the sameness within change (Thyssen & Ceulemans, 2017; Weyl, 1952).

The search for invariants is inherent in the description of reality, introducing dynamic principles of transformation that give meaning to the modern concept of symmetry in science. In other words, a system is said to possess a symmetry if one can make a change (a transformation) in the system so that, after the change, the thing appears exactly the same (is invariant) as before (Lederman & Hill, 2000).

1.3. Research questions

Several notable studies point out that the concepts and the principles of the application of the modern concept of symmetry can be taught and understood at the high school and early college level (Bertozzi, Levrini, & Rodriguez, 2014). Our experimental project tries to answer the following General Research Question:

GRQ. How can we use symmetry and the search of invariants as bridging concepts in science education for fifth grade pupils?

These aims lead to the following Specific Research Questions:

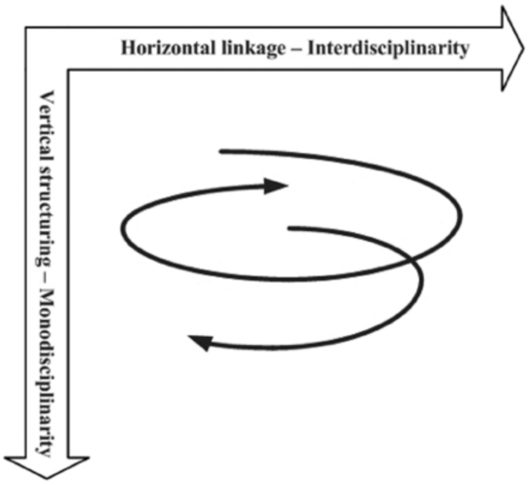
SRQ1. What is the effect of this change of paradigm on the students' understanding and skill development?

SRQ2. What is the effect of this change of paradigm on the didactical framework of linking teaching in Mathematics and Physics?

1.4. Theoretical framework

For modelling activities, centred around symmetry, we are inspired by Michelsen's model (Michelsen, 2015). It consists of an educational framework (Figure 1) for coordination and mutual interaction between mathematics and science. It is operative by two phases: horizontal connection and vertical structuring. In the horizontal phase, thematic integration is established to connect concept and process skills of mathematics and science.

Figure 1.
Michelsen's model: the spiral shape illustrates
the repetitive movements between horizontal and vertical phases.



The horizontal linking is characterized by process of modelling activities in an interdisciplinary context. In the vertical phase, conceptual anchoring of the concepts and process skills from the horizontal phase is used to create languages and symbol systems that allow the pupils to move around logically and analytically within mathematics and science.

According to this model, we have designed the teaching-learning progression considering the vertical increment by developing the concept of symmetry and the transversal dimension by using the sameness within the change to interpret different materials and aspects of these contexts. The vertical and horizontal progression occurs from iterative improvements in which pupils develop the habit of using different articulations of ideas to understand the behaviour of and changes in materials.

Once the concepts and skills in search of invariants are conceptually anchored in pupils at primary school for the respective subjects, they can evolve in a new interdisciplinary context as part of a horizontal linkage. In this way we expect to identify pupils' conceptions and the persistence of these conceptions and to focus the attention on difficulties that children might have in mathematizing physical situations.

2. EXPERIMENT DESIGN

2.1. Classroom-based sample group

A sample of 96 pupils (N=96) meaning that all fifth-grade classes (10 years) of the primary school (I.S.C. "Annibal Caro") in Montegranaro (Italy) participated in the study. The VF and VG classes, in total 41 pupils, were making up the check group that had to answer the same questionnaire at the beginning and at the end of learning and teaching sequence. The inquiry also concerned all the mathematics teachers, who were in charge of answering pre-course and post-course questionnaires and keeping the logbook of the whole experiment.

Table 1.
Scheme of sample group.

Classes	N° Pupils	Gender	Pre-Course Test	Course	Post-Course Test
V A	16	6 F – 10 M	No	Yes	Yes
V C	19	7 F – 12 M	No	Yes	Yes
V D	20	8 F – 12 M	No	Yes	Yes
V F	20	11 F – 9 M	Yes	Yes	Yes
V G	21	10 F – 11 M	Yes	Yes	Yes

2.2. Activities and materials

The set of materials and learning artifacts address providing motivation for students and exhibiting the power of symmetry. The materials and methods should be full of rich pedagogical content and coherent with the educational framework related to symmetries. The design of the educational path is based on the novel of the Little Prince (De Saint-Exupéry, 1945). The use of metaphorical perspective helps to introduce the topic during the first activity. At the same time, it engages the students in making connections between the abstract concept of symmetry and the context of dialogue between Little Prince and the Rose. The metaphor plays a fundamental role in the social and personal constructions of knowledge (Martinez, Sauleda, & Huber, 2001). The structure of 5 activities is divided into three stages of 2 hours in sequence. In total the duration is 6 hours where the researcher proposes various activities and the teachers participate in the lessons as observers.

Table 2.
Series of Activity in TLS.

Activity	Type	Title	Duration
A1	Brainstorming - Writing - Drawing	If I say the word symmetry, what are you thinking about?	2 h
A2	Solving Problems	Boxes and lids	1 h
A3	Listening - Watching	Lesson	1 h
A4	Task Game in group	Strange boxes and lids	2 h
A5	Homework task	Hunting for symmetries.	/

Figure 2.
Activity A1: After the reading of the dialogue between Little Prince and the Rose about Symmetry, pupils write on a post-it note the answer to the following question: Of course, you have already studied symmetry, what do you know about it?

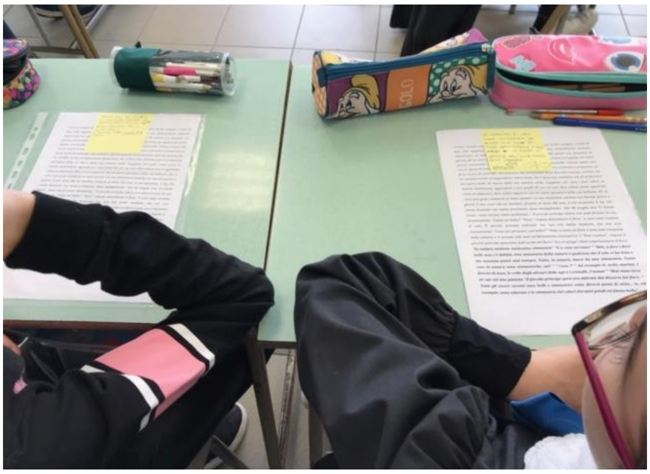
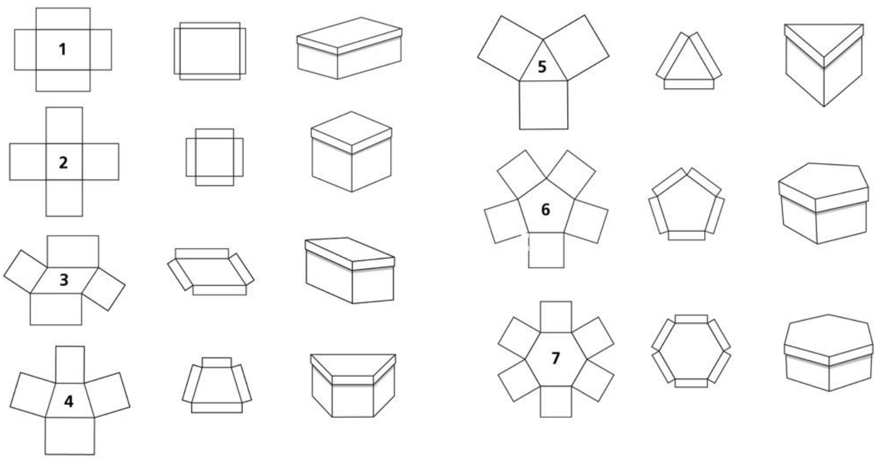


Figure 3.
Activity A4: The pupils, divided into small homogeneous groups of 4/5 pupils chosen by the teacher, will carry out manipulative work following the instructions and filling out the questionnaire of the task game.

The main activity is centered on the task game consisting of closing cardboard boxes, which pupils have to create from the distributed models, and filling out the questionnaire of the task game. This approach implies emphasis on laboratory activities. The manipulative work consists of folding and taping these cardboard models to obtain boxes with their covers. The covers are designed in such a way that they can only be rotated. The task game is to find in how many ways it is possible to put the cover on each box. This game refers to the rotation symmetries of the differently shaped lids.

Figure 4.
Models of cardboard boxes and covers of 7 shapes.



The questionnaire (see Attachment) is made up of four questions of increasing difficulty, linked with the activities carried on during the teaching learning course in order to help the students to propose more accurate and precise answers.

Table 3.
Pupils Questionnaire.

Question	Type	Aim
Q1	Dichotomous Choice Open Justification	To verify the way the concept of symmetry is perceived in the adopted didactical frame.
Q2	8 Items Text	To evaluate and measure the level of knowledge and ability of the tasks and/or specific scopes of the learning unit.
Q3	Narrative Text	D1: Emotional and affective dimension. D2: Cognitive dimension of learning. D3: Possible presence of a cognitive conflict
Q4	A Task Open Justification	To verify to what extent the students are controlling the skills in a (task) exercise of higher cognitive difficulty.

The logical order of the questions is also aimed at stimulating the students in thinking about the whole personal formative process. By asking them to justify their answers, we evaluate the pupils' learning gain and the skills acquired, such as those indicated in the National Guidelines for the Curriculum of the first cycle of education. The time at disposal for answering is of about 45 minutes and it cannot exceed one hour.

3. METHODS

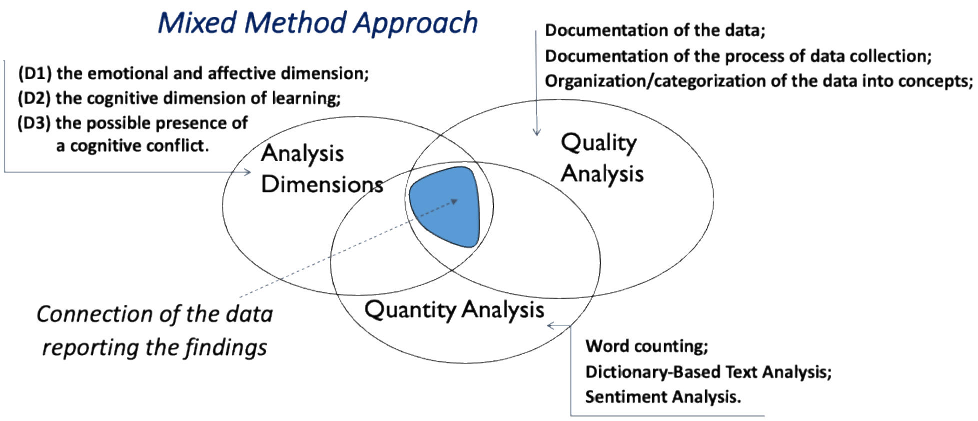
3.1. Research procedure

Research conforms to pre- and post-test design with a control group. The educational project proceeded to implement the TLS on symmetry between the two assessments.

3.2. Data collections and analysis

The task was carried out through multiple levels of development. The research data were selected over several sessions from class observations, audio and video recordings, pupils' drawings and test, and teachers' logbooks and questionaries. Our present work focuses on pupils' pre- and post-test and teachers' logbooks. We assess the effectiveness of the TLS by analysing the learning changes in the level of pupils' knowledge of symmetry resulting from the teaching-learning session. The pupils' answers to the questionnaire with the reflections and observations from teachers' logbooks should trace the effectiveness of teaching tools. The resulting data collection was analysed through a mixed method in conformity with the aim of the study. The study is structured mainly in three domains devoted to cognitive, affective, and psychomotor dimensions. We consider quantitative data by applying appropriate statistico-psychometric models. The investigations incorporate qualitative data with Text Analytics statistics to identify and extract information from pupils' narratives. The multidimensional approach for empirical investigation aims at broadening the level of understanding of the study dimensions and reducing errors of interpretation as much as possible. The connection of the data enhances the trustworthiness of findings.

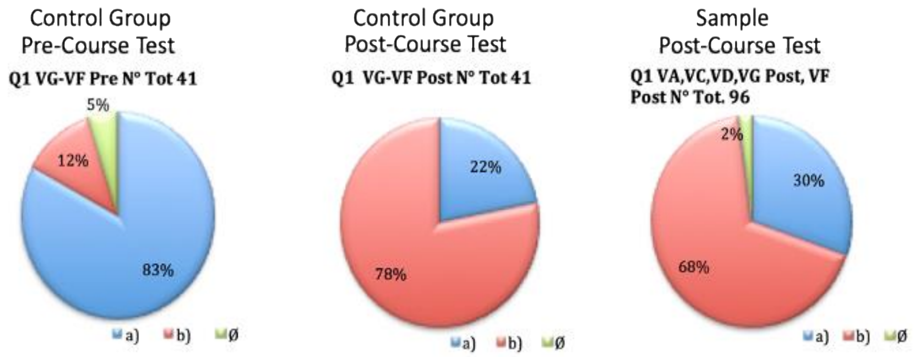
Figure 4.
Integration of different methodological approaches in the workplan analysis of data.



3.3. Findings

In order to answer SRQ1, the analysis of the data shows that the great majority (about 70%) of the students in the sample has chosen the modern concept of symmetry. As for the students of the check group, they too have preferred the modern concept of symmetry (about 80%) in the final questionnaire while having chosen the line symmetry in the initial test (about 83%).

Figure 5.
Comparison of the answer to the question Q1 between pre- and post-test.



Therefore, the experiment seems to have a very positive effect in changing the view of symmetry linked to the search for invariance in the geometric transformations. However, these results are of little relevance if one does not ask how students have interpreted, experienced and assimilated the new concept during the teaching-learning unit. To make a qualitative comparison, we give some relevant examples of answers proposed by the pupils of the VF-VG classes at the beginning and the end of the cycle of lessons.

Pupil VG17, at the beginning, answers the question Q1 in the following way: *"I have chosen the (a) option because it is the most simple and right way to make the Little Prince understand what symmetry is."* In the final questionnaire he offers the following reason of his choice, which at this stage is b: *"I have chosen the (b) option because if one observes his surroundings, one can see that although there are plenty of symmetries, if you turn them it sometimes happens that it is as you wouldn't have."*

Pupil VG19 answers as follows at the beginning: *"I choose (a) because it is clearer than (b)."* She answers at the end *"I choose (b) because if for instance I take a square and I turn it in different ways, the figure remains always the same."*

Pupil VF16 at the beginning: *"I made this choice (a) because it offers a simple and right explanation that makes the Little Prince understand what symmetry is."* At the final test: *"I have chosen (b) because it is true that there exist so many symmetries, as the rotation, that transform a figure, and this transformed figure is really coincident with the original one."*

In order to answer SRQ2, we have picked out the most frequent keywords, the most recurrent and significant arguments that appear in the justifications in order to classify them in thematic groups according to the aims of the TLS. The expressions *"many symmetries"*, *"movements"*, *"transformations"*, *"palindrome"*, *"coincidence with the original"* occurring in the answers to the final questionnaire are almost absent in the answers to the initial one.

This result means that the TLS helped the students to take an active part in acquiring a new lexicon and also that they became able to cognitively elaborate, at least to some extent, the new concepts, and to review the concepts they had already acquired. The ability of some pupils to shape the experience can sometimes result in metacognitive skills developing original and unexpected explanations. Very interesting is the answer of VF4, which shows the ability to extend the meaning of symmetry because VF4 interprets the palindromic characteristic of the short film entitled *"Palindromic Film"* by association of thought with the argument a certain symmetry of time. The pupils' answers to the fourth question (Q4) reveal an improved mastery of skills in the exercise of higher cognitive difficulty. Data show that 55% of the students in the sample (about 53/96) have given a correct and pertinent answer to the question. The pupils in the check group gave a correct answer for 27% (about 11/41) to the initial questionnaire while 51% of them (about 21/41) to the final questionnaire, which constitutes an important increase (about 25%).

Table 4.
Data from answers to the fourth question (Q4).

	N° Students	Pertinent A.	Not Pertinent A.	Confused A.	Not Answered
VF-VG Pre	41	11 (27%)	28 (68%)	0 (0%)	2 (5%)
VF-VG Post	41	21 (51%)	20 (49%)	0 (0%)	0 (0%)
Whole Sam.	96	53 (55%)	36 (38%)	4 (4%)	3 (3%)

The effectiveness of the educational activities is recognized not only in the knowledge and skills transferred, but also in the positive and authentic attitudes towards a training process of integration and innovation of the operational and abstract meaning of symmetry in the entire educational context. In this regard, the VF-VG class teacher shows that she feels *"more confident and trained"* in the contents and teaching practices *"thanks to the explanations and reflections made with the teacher"* during the activities.

The project offered broad space for reflection and renegotiation of the consolidated pedagogical-educational practices on the concept of symmetry (e.g., folding papers, mirror images) and new stimuli for future educational experiments. In her logbook, the teacher reports that *"the project turned out to be an excellent incisive training course for her personal experience."* These results bring into light that the teaching learning sequence (TLS) constitutes a promising step along the path to application of the modern concept of symmetry.

4. CONCLUSIONS

The data analysis shows that the pupils were in the regime of competence for grasping the changed view of symmetry linked to the search for invariance. Nevertheless, it does not permit to evaluate how deeply the students have elaborated the contents nor how conscious they were of the whole process. However, the triangulation of information derived from different assessment methods suggests that students can elaborate on the experience. It also provides fundamental ideas that symmetry and invariance concepts will allow building the architecture of more extensive scientific knowledge. The evidence shows that manipulation game experience grants intense spatial reasoning work besides the vision of line symmetry associated with the traditional materials.

Diversifying the consolidated symmetry application gives interesting information in the students' cognitive structure to build abstract and formal knowledge. The opportunities to apply representations and challenging them to explain phenomena in different descriptions can lead to a more reaching learning scientific and mathematical concepts inherent to symmetry. Thus, the study of transformations and invariants facilitates in the pupils the acquisition of cognitive procedures and processes that can be extended to many areas, modelling and generalization processes. Based on such consideration, it would be interesting to implement combined symmetries of different designs and movements associated with new covers of cardboard boxes. For this reason, we plan to carry on with the project regarding the results of the present work.

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ENDNOTE

1 The principle of legality or regularity registered at the level of nature, physical constants or specific natural properties points towards the existence of "forms" in nature. The transformations in the material world refer not only to an "efficient causality" but also, necessarily, to a "formal causality". It is explicitly demonstrated that some physics laws not only reflect such order but can be derived directly from it (Knuth, 2016).

ATTACHMENT

Pupils Questionaries

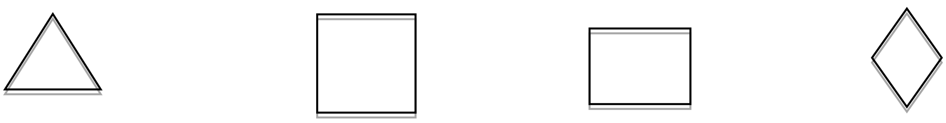
Q1) If you were the "Rose" trying to explain symmetry to the "Little Prince", which of the following sentences would you use?

a)There is symmetry when you can see that, if a line divides a figure in two parts, these parts reflect each other like in a mirror.

b)Among the many symmetries that exist there are also the movements that transform a figure so that the resulting figure coincides with the original.

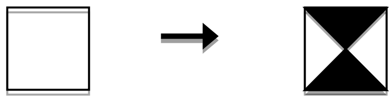
Write the reason of your choice.

Q2) After you helped Paul in closing boxes of various forms, write in how many ways it is possible to put the cover on each box, and draw the symmetry lines (axis), if any.



Q3) Tell the "Little Prince" the activities carried out in the classroom, what you liked, what you learned the most about symmetries and if you had any difficulties.

Q4) The “Rose” told the “Little Prince” that there exist also colour symmetries. By decorating a figure with colours, one can change its symmetry. Do you agree? Explain what happens to the symmetry of the square when from completely white it becomes coloured in the following way.



AUTHORS’ INFORMATION

Full name: Simone Brasili
Institutional affiliation: University of Camerino, Italy
Institutional address: Via Madonna delle Carceri 9, 62032, Camerino
Short biographical sketch: PhD Candidate in Science, Technology and Mathematics at University of Camerino. His research interest relates to physics and mathematics education, including topics about interdisciplinarity between Physics and Mathematics.

Full name: Riccardo Piergallini
Institutional affiliation: University of Camerino, Italy
Institutional address: Via Madonna delle Carceri 9, 62032, Camerino
Short biographical sketch: Full Professor of Geometry at University of Camerino. His research field is low dimensional topology. He is also interested in educational and application aspects of geometry and topology.